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Adomian Decomposition Method and The Other Integral Transform

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Abstract

The Adomian decomposition method is an iterative method that can be used to solve integral, differential, and integrodifferential equations. The differential equations that can be solved by this method can be of integer or fractional order, ordinary or partial, with initial or boundary value problems, with variable or constant coefficients, linear or nonlinear, homogeneous or nonhomogeneous. This method divides the equation into two forms, namely linear and nonlinear, so that it can solve equations without linearization, discretization, perturbation, or other restrictive assumptions. The basic concept of this method assumes that the solution can be decomposed into an infinite series. This method decomposes the nonlinear form (if any) of the equation with the Adomian polynomial series. This decomposition method can be combined with various integral transform, such as Laplace, Sumudu, Elzaki, and Mohand. The main idea of this technique assumes that the solution can be decomposed into an infinite series, then applies the integral transform to the differential equation. The main advantage of this technique is that the solution can be expressed as an infinite series that converges rapidly to the exact solution. This paper aims to combine the Adomian decomposition method with the new integral transform introduced by Kumar et al. (2022). This integral transform is called the Rishi transform. A scheme for solving fractional ordinary differential equations using the combined method is presented in this paper.

Keywords: Adomian decomposition method, Rishi transform, fractional ordinary differential equation.

1. Introduction

The Adomian decomposition method was first introduced by George Adomian to solve a system of stochastic equations (Adomian, 1980). This method can be an effective procedure to obtain analytical solutions without linearization or weak nonlinear assumptions, perturbation, discretization, or restrictive assumptions in stochastic cases (Adomian, 1988). The Adomian decomposition method can be used to solve integral, differential, and integral-differential equations. Differential equations that can be solved by this decomposition method can be of integer or fractional order, ordinary or partial, with initial or boundary value problems, with variable or constant coefficients, linear or nonlinear, homogeneous or nonhomogeneous. This decomposition method is also able to solve algebraic equations, delay differential equations, and equation systems (Duan et al., 2012; Al-awawdah, 2016; Sumiati et al., 2019).

The basic concept of this method assumes that the solution is decomposed into an infinite series, the nonlinear form is decomposed into Adomian polynomials, and an iterative algorithm is constructed to recursively determine the solution. The Adomian decomposition method is a powerful and useful technique for solving heat equations (Biazar & Amirtaimoori, 2005), and waves (Luo et al., 2006). Combined with the Caputo derivative, the Adomian decomposition method can solve diffusion differential equations, waves (Jafari & Daftardar-Gejji, 2006), and Burgers (Gepreel, 2012) with fractional order.

The numerical scheme for the Laplace transform based on the Adomian decomposition method can be used to obtain an approximate solution to nonlinear differential equations. The main idea of this technique assumes that the solution can be decomposed into an infinite series, then applies the Laplace transform to the differential equation. The main advantage of this technique is that the solution can be expressed as an infinite series that converges rapidly to the exact solution (Khuri, 2001). The Adomian-Laplace decomposition method is used for solving the Volterra integrodifferential equation (Wazwaz, 2010), non-homogeneous heat equations that arise in fractal heat flow (Jassim, 2015), and fractional Black-Scholes European option pricing equation (Owoyemi et al., 2020).

Other integral transforms that can be combined with the Adomian decomposition method are the Sumudu (Khan et al., 2008; Mahdy & Marai. 2018), Elzaki (Elzaki & Alkhateeb, 2015; Mohamed & Elzaki, 2020; Sumiati et al., 2020a), and Kashuri-Fundo (Sumiati et al., 2020b; Subartini et al., 2021).

According to Kumar et al. (2022), Rishi transform is better than other existing integral transforms because it gives the exact result of the problem without doing tedious computational work and spending little time. Rishi transform has a duality relationship with Laplace transform. This relationship makes Rishi transform valuable because all the properties of Laplace transform are achieved.

Therefore, based on the background of the problems and previous studies that have been presented, this paper aims to combine the Adomian decomposition method with the new integral transform, that is Rishi transform. A schematic for solving ordinary differential equations with fractional order using the combined method is presented in this paper.

2. Literature Review

This section presents the basic theories and concepts related to fractional calculus and Rishi transform.

Definition 1. (Kumar et al., 2022) The Rishi transform of exponential order piecewise continuous function, $\omega(t)$ defined in the interval $[0,\infty)$ is given by

$$\mathcal{R}[f(t)] = T(\varepsilon,\sigma) = \frac{\sigma}{\varepsilon}\int_0^\infty f(t)e^{-\left(\frac{\varepsilon}{\sigma}\right)t}dt\,, \varepsilon \geq 0, \sigma > 0.$$

Next, the inverse of Rishi transform is denoted by $\mathcal{R}^{-1}[T(\varepsilon,\sigma)] = f(t)$, $t \geq 0$. For α is a fractional number, valid

$$\mathcal{R}[t^\alpha] = \left(\frac{\sigma}{\varepsilon}\right)^{\alpha+2}\Gamma(\alpha+1). \quad (1)$$

Definition 2. (Podlubny, 1999; Mathai & Haubold, 2017) The Caputo fractional derivative of the function f with t in the order α, where $\alpha > 0$, is defined as

$${}_a^C D_t^\alpha f(t) = \frac{1}{\Gamma(n-\alpha)}\int_a^t (t-u)^{n-\alpha-1}f^{(n)}(u)du\,, n-1 < \alpha \leq n.$$

Definition 3. (Kumar et al., 2020) The Rishi transform of Caputo fractional derivative for $a = 0$ is defined as

$$\mathcal{R}[{}^C D_t^\alpha f(t)] = \left(\frac{\varepsilon}{\sigma}\right)^\alpha T(\varepsilon,\sigma) - \sum_{k=0}^{n-1}\left(\frac{\varepsilon}{\sigma}\right)^{\alpha-k-2} f^{(k)}(0)\,, n-1 < \alpha \leq n.$$

3. Rishi Decomposition Method

This section presents the solution of fractional ordinary differential equations using the Rishi decomposition method.

Given the fractional ordinary differential equation as follows

$$D_t^\alpha y(t) = g(t) + Ny(t) + Ry(t), \quad (2)$$

and initial condition $y(0) = c$, where $D_t^\alpha \equiv {}^C D_t^\alpha$ is a Caputo fractional derivative operator with $0 < \alpha \leq 1$, N is a nonlinear operator, R is a linear operator, g is a function that shows the homogeneity of the differential equation, and y is a function of t to be determined. Using the Rishi transform in equation (2), thus based on Definition 3, is obtained

$$y(\varepsilon,\sigma) = \left(\frac{\sigma}{\varepsilon}\right)^2 y(0) + \left(\frac{\sigma}{\varepsilon}\right)^\alpha \mathcal{R}[g(t)] + \left(\frac{\sigma}{\varepsilon}\right)^\alpha \mathcal{R}[Ny(t)] + \left(\frac{\sigma}{\varepsilon}\right)^\alpha \mathcal{R}[Ry(t)]. \quad (3)$$

Next, using the inverse of Rishi transform in equation (3), is obtained

$$y(t) = y(0) + \mathcal{R}^{-1}\left[\left(\frac{\sigma}{\varepsilon}\right)^\alpha \mathcal{R}[g(t)]\right] + \mathcal{R}^{-1}\left[\left(\frac{\sigma}{\varepsilon}\right)^\alpha \mathcal{R}[Ny(t)]\right] + \mathcal{R}^{-1}\left[\left(\frac{\sigma}{\varepsilon}\right)^\alpha \mathcal{R}[Ry(t)]\right]. \quad (4)$$

The Adomian decomposition method assumes that the y function can be broken down or decomposed into an infinite series (Adomian, 1988; Al Awadah, 2016)

$$y(t) = \sum_{n=0}^\infty y_n(t) = y_0 + y_1 + y_2 + \cdots, \quad (5)$$

where y_n can be specified recursively. This method also assumes the nonlinear operator Ny can be decomposed into an infinite polynomial series

$$Ny = \sum_{n=0}^\infty A_n, \quad (6)$$

where $A_n = A_n(y_0, y_1, y_2, \dots, y_n)$ is a defined Adomian polynomial,

$$A_n(y_0, y_1, y_2, \dots, y_n) = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{k=0}^{n}\lambda^k y_k\right)\right]_{\lambda=0}, n = 0,1,2,\dots$$

where λ is a parameter.

Substitute equations (5) and (6) to equation (4), obtained

$$\sum_{n=0}^{\infty} y_n(t) = y(0) + \mathcal{R}^{-1}\left[\left(\frac{\sigma}{\varepsilon}\right)^{\alpha}\mathcal{R}[g(t)]\right] + \mathcal{R}^{-1}\left[\left(\frac{\sigma}{\varepsilon}\right)^{\alpha}\mathcal{R}\left[\sum_{n=0}^{\infty} A_n\right]\right] + \mathcal{R}^{-1}\left[\left(\frac{\sigma}{\varepsilon}\right)^{\alpha}\mathcal{R}\left[R\sum_{n=0}^{\infty} y_n(t)\right]\right]. \quad (7)$$

If both sides of equation (7) are described, then successively is obtained

$$y_0 = y(0) + \mathcal{R}^{-1}\left[\left(\frac{\sigma}{\varepsilon}\right)^{\alpha}\mathcal{R}[g(t)]\right],$$
$$y_1 = \mathcal{R}^{-1}\left[\left(\frac{\sigma}{\varepsilon}\right)^{\alpha}\mathcal{R}[A_0]\right] + \mathcal{R}^{-1}\left[\left(\frac{\sigma}{\varepsilon}\right)^{\alpha}\mathcal{R}[Ry_0]\right],$$
$$y_2 = \mathcal{R}^{-1}\left[\left(\frac{\sigma}{\varepsilon}\right)^{\alpha}\mathcal{R}[A_1]\right] + \mathcal{R}^{-1}\left[\left(\frac{\sigma}{\varepsilon}\right)^{\alpha}\mathcal{R}[Ry_1]\right],$$
$$y_3 = \mathcal{R}^{-1}\left[\left(\frac{\sigma}{\varepsilon}\right)^{\alpha}\mathcal{R}[A_2]\right] + \mathcal{R}^{-1}\left[\left(\frac{\sigma}{\varepsilon}\right)^{\alpha}\mathcal{R}[Ry_2]\right],$$
$$\vdots$$

thus generally obtained the recursive relation of the fractional ordinary differential equation solution (2) using the Rishi decomposition method as follows

$$\begin{aligned} y_0 &= y(0) + \mathcal{R}^{-1}\left[\left(\frac{\sigma}{\varepsilon}\right)^{\alpha}\mathcal{R}[g(t)]\right], \\ y_{n+1} &= \mathcal{R}^{-1}\left[\left(\frac{\sigma}{\varepsilon}\right)^{\alpha}\mathcal{R}[A_n]\right] + \mathcal{R}^{-1}\left[\left(\frac{\sigma}{\varepsilon}\right)^{\alpha}\mathcal{R}[Ry_n]\right], n = 0,1,2,\dots. \end{aligned} \quad (8)$$

Therefore, the approximate solution of the fractional ordinary differential equation (2) using the Rishi decomposition method is

$$y \approx \sum_{n=0}^{k} y_n, \text{where } \lim_{k\to\infty}\sum_{n=0}^{k} y_n = y.$$

4. Conclussion

The Rishi decomposition method is a combination of the Adomian decomposition method and the Rishi integral transform. This paper presents a scheme for solving fractional ordinary differential equations using the Rishi decomposition method.

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