



Value-at-Risk Estimation of Indofood (ICBP) and Gas Company (PGAS) Stocks Using the ARMA-GJR-GARCH Model

Herlina Napitupulu¹, Rizki Apriva Hidayana^{2*}, Jumadil Saputra³

¹*Department of Mathematics, Faculty of Mathematics and Natural Sciences, Padjadjaran University, Jatinangor, West Java, Indonesia*

²*Master's Program of Mathematics, Faculty of Mathematics and Natural Sciences, Padjadjaran University, Jatinangor, West Java, Indonesia*

³*Faculty of Business, Economics and Social Development, Universiti Malaysia Terengganu, 21030 Kuala Nerus, Terengganu, Malaysia*

*Corresponding author email: rizki20011@mail.unpad.ac.id

Abstract

Stocks are one of the most widely used financial market instruments by investors in investing. The most important component of any investment is volatility. Volatility is a conditional measure of variance in stock returns and is important for risk management. In addition to volatility, the important things in investing are return and risk. Risk can be measured using Value-at-Risk (VaR) and can estimate the maximum loss that occurs. The purpose of this study is to determine VaR using the Autoregressive Moving Average-Glosten Jagannathan Runkle-Generalized Autoregressive Conditional Heteroscedasticity (ARMA-GJR-GARCH) model. The stages of data analysis used are estimating the ARMA model and the GARCH model, then estimating the GJR-GARCH model by looking at the heteroscedasticity and asymmetric effects on the GARCH model. Next, determine the VaR value from the estimated mean and variance (volatility) using the ARMA-GJR-GARCH model. The results of the model estimator obtained are based on the return data for the four stocks analyzed, namely the ARMA (5,5)-GJR-GARCH (1,1) model for ICBP stocks and ARMA (1,2)-GJR-GARCH (1,1) for PGAS shares. The Value-at-Risk values of each stock are 0.060427 and 0.024724. This research can be used by investors as a consideration in making investment decisions.

Keywords: Return, volatilitas, Value-at-Risk, ARMA-GJR-GARCH

1. Introduction

Investment is an activity of placing funds that are carried out at this time in the hope of obtaining benefits in the future. In the object of investment, assets are generally divided into two, namely real assets and financial assets. Real assets are related to infrastructure and financial assets are related to stocks (Dwipa, 2016). Investors choose to invest in shares in a company based on the desire to gain profits in the future which can be seen from the number of stock returns. Investing in stocks is faced with risk because stock returns are volatile. Stock returns can change very quickly, so the stock index value can also change, this movement is known as stock return volatility. High volatility results in high risk, as well as low volatility results in low risk. Therefore, it needs to be overcome by using a mathematical model.

Several researchers have used various time series models, one of which is the ARCH model introduced by Engle in 1982. According to Dwipa (2016) stock returns have three characteristics. The first is the grouping of volatility, meaning that very large changes occur at certain times and small changes in other periods. The second is fat-tailedness (excess kurtosis) which means that stock returns display a sharpness that is greater than the normal distribution. Third, there is a leverage effect, which is a condition where bad news and good news have an asymmetric effect on return volatility.

The Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model has a more flexible structure to accommodate the volatility in Bollerslev's (1986) stock. Tamilselvan and Vali (2016) used the GARCH model in examining Muscat stocks on the security market by concluding that the GARCH (1,1) model is the best estimate for symmetric data and there is no leverage effect on the data used. However, the GARCH model cannot be used on data that has a leverage effect. Ali (2013) used the EGARCH, GJR-GARCH, TGARCH, AVGARCH, NGARCH, IGARCH, and APARCH models to determine the functional relationship of the time series of pathogenic indicators to

activate reactions on the coast. However, the TGARCH model is marginally better than other models in capturing the response of the pathogenic variable indicator. Mittnik, Paolella, and Rachev (2002) investigated the stationarity of the stable GARCH process. However, the process in the GARCH model cannot explain the asymmetric phenomenon, therefore the Glosten Jagannathan Runkle-Generalized Autoregressive Conditional Heteroscedasticity (GJR-GARCH) model will be used. The GJR-GARCH model can overcome the asymmetric effect (Lee, 2007).

Measurement of risk in this study using Value at Risk. According to Bakhtiar *et al.* (2020), Value at Risk is one of the most popular tools used by investors in measuring risk. Value at Risk is used as a measuring tool that can assess the worst losses in investing at a certain time and level of confidence. Several studies on the level of risk measurement use Value at Risk. Sukono *et al.* (2019) examine the ARIMA-GARCH model which is used to estimate and expect a shortfall of several stocks in the Indonesian capital market. Based on the analysis, selected stocks are obtained. Bank Mandiri's shares have the lowest risk level and Mustika Ratu's shares have the highest level of risk with the value-at-risk of the stock generally smaller than the expected shortfall value. Bucevska (2012) conducted a relative test of the selected GARCH-type model in terms of the ability to estimate volatility and extended empirical research on VaR estimation in financial markets. Nilsson (2017) found the best APARCH model for estimating volatility, while for estimating VaR the best model is APARCH, GJR-GARCH, or EGARCH depending on which VaR level is used.

From the research that has been described, there are still some shortcomings in these models. Among others, Tamilselvan and Vali (2016) examined Muscat stocks using the GARCH model on symmetric data, but if the data is asymmetrical the GARCH model cannot be used. One solution for asymmetric data is the GJR-GARCH model. Sukono *et al.* (2019) have determined VaR using the time series model but have not used the GJR-GARCH model.

Based on this gap, this study uses the Autoregressive Moving Average-Glosten Jagannathan Runkle-Generalized Autoregressive Conditional Heteroscedasticity (ARMA-GJR-GARCH) model to determine the Value at Risk value on stock returns. The purpose of this study is to determine Value at Risk using the ARMA-GJR-GARCH model on stock return data. The hope of this research is that it can be used by investors to help make investment decisions.

2. Literature Review

2.1. Return

According to Ruppert (2011) return is the rate of return on the results obtained as a result of investing. In general, the formula for return is as follows:

$$r_t = \ln \left(\frac{S(t_i)}{S(t_{i-1})} \right) \quad (1)$$

where r_t is stock return at time t , $S(t_i)$ is the stock price in period t_i and $S(t_{i-1})$ is the stock price at period t_{i-1} .

2.2. Stasianarity

The stationarity test is the underlying assumption in the statistical procedures used in time series analysis. According to Tsay (2005), the data stationarity test can use the Augmented Dickey-Fuller (ADF) test. The Augmented Dickey-Fuller (ADF) test is a stationary test on average, where the statistics for the ADF test are as follows:

$$ADF = \frac{\hat{\delta}}{SE(\hat{\delta})} \quad (2)$$

where $SE(\hat{\delta})$ is the standard error for $\hat{\delta}$. The decision-making criteria are if the $ADF < \alpha$ then reject H_0 in other words, the data is stationary. If the ADF value $> \alpha$ then accept H_0 in other words, the data is not stationary.

2.3. ARMA Model

The purpose of the ARMA model is to discuss the average model in the time series. In general, the Autoregressive Moving Average (ARMA) model can be expressed in the following equation:

$$r_t = \omega + \sum_{i=1}^p \phi_i r_{t-i} + a_t - \sum_{j=1}^q \theta_j a_{t-j} \quad (3)$$

where r_t is the return value at time t , a_t is a white noise process or error at time t (Susanti *et al.*, 2017).

ARMA modeling process. In general, the ARMA modeling process is: (i) Model identification by determining p and q values with autocorrelation function (ACF) and partial autocorrelation function (PACF) from correlogram plots. (ii) Parameter estimation can use the method of least squares or maximum likelihood. (iii) Diagnostic test with white noise and non-correlation test on residuals using Box-Pierce or Ljung-Box. (iv) Forecasting, if the model is suitable it can be used for recursive predictions.

2.4. GARCH Model

Bollerslev (1986) developed the ARCH model into a GARCH model (p, q) where q is the order ARCH and p is the order GARCH. In general, the GARCH model is as follows:

$$a_t = \sigma_t \varepsilon_t, \quad \sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i a_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2 \quad (4)$$

where ε_t is the order of independent and identically distributed (iid), σ_t^2 is the residual variance at time $-t$, ω is the constant component, α_i is the i -th parameter of ARCH, a_{t-i}^2 is the square of the residual at a time to $(t-i)$, β_j is the j -th parameter of GARCH, σ_{t-j}^2 is the variance of the residual at a time to $(t-j)$. Equation (3) shows that the conditional variance is a past shock as seen from the squared residual (p) and the past residual variance (q) (Olwe and Ayodeji, 2010).

The GARCH modeling process. In general, the volatility modeling process is: (i) Estimated ARMA model with time series model. (ii) Use the residuals from the ARMA model to test the ARCH effect. (iii) If there is an ARCH effect, the estimation of the volatility model, and the combined estimates form the ARMA model and the volatility model. (iv) Perform diagnostic tests to observe the suitability of the model. (v) If the model already fits, use it to predict based on recursive predictions.

2.5. Asymmetry

According to Bakhtiar et al. (2020) the asymmetric test is a property that shows an imbalance in certain conditions or objects. In the time series, the asymmetric nature is called the leverage effect or high volatility. To determine the nature of asymmetry, namely by skewness and kurtosis. Skewness is a degree of imbalance in the distribution. The asymmetric test can be done by using the cross-correlation between the lag residual (ε_t) and the squared residual (ε_t^2).

According to Brook (2008) in general, asymmetric testing can be tested by testing the sign bias in the regression equation as follows:

$$\hat{u}_t^2 = \phi_0 + \phi_1 S_{t-1}^- + \phi_2 S_{t-1}^- u_{t-1} + \phi_3 S_{t-1}^+ u_{t-1} + \nu_t \quad (5)$$

where S_{t-1}^- is a dummy indicator with a value of 1 if $\hat{u}_{t-1} < 0$ and 0 for others, ν_t is an error, $S_{t-1}^+ = 1 - S_{t-1}^-$ which is a positive observation, ϕ_1 is a sign bias parameter (positive or negative effect), ϕ_2 and ϕ_3 is a size bias parameter.

Equation (7) can be used to see the effect of asymmetry on the model by looking at the probability value of the dummy multiplication with the residual GARCH model. Reject H_0 if the probability value of the dummy multiplication with the GARCH model residual $< \alpha$ then the residual is asymmetric.

2.6. GJR-GARCH Model

Glosten Jagannathan and Runkle (1993) introduced the asymmetric GARCH model, namely the GJR-GARCH model. The advantage of the GJR-GARCH model is that it can measure volatility due to the different effects of bad news and good news. The difference between the GJR-GARCH model and the GARCH model is that the GJR-GARCH model has parts that represent asymmetric properties. The GJR-GARCH model is as follows:

$$a_t = \sigma_t \varepsilon_t, \quad \sigma_t^2 = \omega + \sum_{i=1}^p \alpha_i a_{t-i}^2 + \sum_{j=1}^p \beta_j \sigma_{t-j}^2 + \gamma_i I_{t-i} a_{t-i}^2 \quad (6)$$

and

$$I_{t-i} = \begin{cases} 1, & a_{t-i} < 0 \\ 0, & a_{t-i} \geq 0 \end{cases} \quad (7)$$

where α_i is the parameter to i of ARCH, β_j is the parameter to j of GARCH and γ_i is the parameter to i of the leverage effect, I_{t-i} is a dummy variable which means a functional index that is zero when a_{t-i} positive and one when a_{t-i} negative. If the parameter $\gamma_i > 0$ then the negative error does not work, which means that the effect of bad news will be greater than the effect of good news (Dritsaki, 2017).

GJR-GARCH modeling process: (i) Estimation of GARCH model with time series model. (ii) Use the residuals from the GARCH model to test the ARCH effect. (iii) Conducting diagnostic tests to observe the suitability of the model. (iv) Test for asymmetric effects. (v) If there is an asymmetric effect, it can be used to predict based on recursive prediction.

2.7. Value-at-Risk

According to Dwipa (2016), VaR is defined as the maximum potential loss in a certain period with a certain level of confidence in normal (market) conditions. VaR at the confidence level $(1 - \alpha)$ and time interval t can be formulated as follows:

$$VaR = \inf\{r_t | F_l(r_t) \geq \alpha\} \quad (8)$$

where F_l is the distribution function of return r_t . Then the VaR for the next period with a confidence level of α can be formulated as follows:

$$VaR = \mu + \sigma_t F^{-1}(\alpha) \quad (9)$$

where μ is the mean, σ^2 is the variance, and σ is the standard deviation.

2.8. Backtesting

Back-test is a method used to measure the estimated VaR performance. Suppose r_t represents gain or loss at time t and VaR_t is a prediction of VaR at time t . In 1998 Lopez introduced a model of the size-adjusted frequency approach as follows:

$$C_t = \begin{cases} 1 + (r_t - VaR_t)^2, & r_t > VaR_t \\ 0, & r_t \leq VaR_t \end{cases} \quad (10)$$

The statistic used to test the VaR risk performance is by using the quadratic probability score (QPS). The QPS equation is as follows:

$$QPS = \left(\frac{2}{n}\right) \sum_{i=1}^n (C_t - p)^2 \quad (11)$$

where n is the number of data, p is the probability value. The QPS value is between the range $[0, 2]$ where 0 is the minimum value that occurs when $r_t \leq VaR_t$ and 2 is the maximum value that occurs when $r_t > VaR_t$. VaR performance is said to be good when the small QPS is close to 0 (Sukono et al., 2019)

3. Results and Discussion

3.1. Data Return

The data used is secondary data on the closing prices of ICBP and UNVR shares between 17 December 2018 to 14 December 2021 and obtained from the website <https://finance.yahoo.com/>. The software used in this research is Eviews 10 and Ms. Excel. The time series model used in this study is the ARMA-GJR-GARCH model using stock return data, namely ICBP shares, and PGAS shares. The closing price data is used to calculate the return value using equation (1). So, by using the Eviews 10 software, the return value for each stock is very volatile and forms a cluster where there are relatively high clusters and relatively low clusters. Furthermore, testing the stationarity of stock returns.

3.2. Stationarity Test

Stationary test using Dickey Fuller with a probability value of 5%. The formula used for this stationary test is by using equation (2). Stationary testing in this study using Eviews 10 software, so the stationarity test results are given in Table 1.

Table 1. Stock return stationary test

Code	ADF value	Critical Value	Probability	Stationarity
ICBP	-28.91462			
PGAS	-25.62396	-2.8652	0.0000	Stationary

Based on the stationary test in Table 1, the probability value obtained is less than the probability value used, which is 5%, meaning that the ICBP and PGAS stock return data are stationary.

3.3. ARMA Model and Diagnostic Test

Stock return data that has been stationary will be continued by identifying the ARMA model. The ARMA model can be identified by looking at the ACF and PACF on the correlogram plot. Based on Table 1, there is no differencing process for ICBP and PGAS stock data because the two stocks are stationary at the level. The modeling process uses equation (3), so the model for ICBP stock is ARMA (5,5) and PGAS stock ARMA (1,2). The model is given as

follows:

$$\text{ICBP stock: } z_t = -1.047400z_{t-1} - 1.061424z_{t-2} - 1.046466z_{t-3} - 1.050977z_{t-4} - 0.074621z_{t-5} - 0.980362a_{t-5} + a_t$$

$$\text{PGAS stock: } z_t = -0.944571z_{t-1} - 0.929149a_{t-2} + a_t$$

After obtaining the ARMA model, then the significance test of the above model. Decision-making was seen from the *p-value*. If *p-value* > α then H_0 accepted and H_1 rejected, if *p-value* < α then H_0 rejected and H_1 accepted. The results of the t-test were obtained in the above model with a probability value of <0.05, meaning that the lag obtained in each model has a significant effect on stock return data. After that, a diagnostic test is carried out on the model to see that the model obtained is suitable for use. The diagnostic test used in these models is that both models have white noise.

3.4. GARCH Modeling and Diagnostic Test

Before doing volatility modeling first, we have to check the effect of heteroscedasticity using Eviews 10 software. So that the ARMA model has an ARCH effect and can be continued to identify the GARCH model. GARCH estimation by looking at the ACF and PACF plots and the modeling process is carried out using equation (4). The results of the GARCH modeling can be seen in Table 2.

Table 2. GARCH Model

Code	GARCH Model	Model Equation
ICBP	GARCH (1,1)	$\sigma_t^2 = 8.10 \times 10^{-5} + 0.220051a_{t-1}^2 + 0.511680\sigma_{t-1}^2 + u_t$
PGAS	GARCH (1,1)	$\sigma_t^2 = 4.28 \times 10^{-5} + 0.091825a_{t-1}^2 + 0.863369\sigma_{t-1}^2 + u_t$

Based on the results in Table 2, the two models were tested for significance on the model using a diagnostic test. The results of the diagnostic test on the GARCH (1,1) model for each stock show that the model has white noise. So the GARCH (1,1) model for the two stocks used can be continued to the next stage, namely the asymmetric effect test.

3.5. Asymmetry Test

This asymmetric test is also known as the cross-correlation test, which means the multiplication between the lag residuals (u_t) with squared residual (u_t^2). The multiplication is carried out to see whether the GARCH model in Table 2 has asymmetric properties or not. To check the cross-correlation using Eviews 10 software and using equation (5). The results obtained for the four models are that there is no value equal to zero, so the data used has an asymmetric nature, then the conditions of bad news and good news have an asymmetric effect on volatility.

3.6. GJR-GARCH Model

GJR-GARCH modeling can be done if you already know the asymmetric nature of the GARCH model contained in Table 2. The parameter estimation of the GJR-GARCH model is carried out using equation (6) with the help of Eviews 10 software. The GJR-GARCH (1,1) model is given as follows:

$$\text{ICBP stock: } \sigma_t^2 = 7.12 \times 10^{-5} + 0.078465a_{t-1}^2 + 0.189747a_{t-1}^2I_{t-1} + 0.581342\sigma_{t-1}^2 + \varepsilon_t$$

$$\text{PGAS stock: } \sigma_t^2 = 4.31 \times 10^{-5} + 0.063100a_{t-1}^2 + 0.063350a_{t-1}^2I_{t-1} + 0.864486\sigma_{t-1}^2 + \varepsilon_t$$

Based on the above model, the asymmetric value obtained from each of these models is not equal to zero, which means that there is a shock so that the volatility of the return value for the leverage effect has a significant effect. The results of the two models have a greater influence on bad news received in return volatility than on good news.

3.7. Value-at-Risk and Backtesting

Before determining the Value-at-Risk value, the average value and volatility of stock returns for the next period are predicted. Using the average model and stock return volatility, the value is calculated $\hat{\mu}_t = \hat{r}_t$ and $\hat{\sigma}_t^2 = \sigma_t^2$. These results can be seen in Table 3.

Table 3. Average, variance, VaR and QPS

Code	$\hat{\mu}_t$	$\hat{\sigma}_t^2$	$\hat{\sigma}_t$	Var_t	QPS
ICBP	-0.001327	0.001291	0.035930	0.060427	0.012248
PGAS	0.000811	0.000241	0.015524	0.024724	0.292517

The Value-at-Risk value is determined based on the results of the ARIMA model for the average and GJR-GARCH for volatility (variance). Estimates of the average $\hat{\mu}$ and the volatility (variance) σ^2 are in Table 3. If the probability value is 5%, then the normal distribution value $z_{0.05} = -1.65$ and the investment assumption is $S_0 = 1$ unit, then the Value-at-Risk value is obtained using equation (9) and the results are according to Table 5 in column VaR_t .

Value-at-Risk performance estimates are evaluated using backtesting. If the probability value used is 5%, then by using equation (10) and equation (11) the QPS results are obtained as in Table 3 QPS column. The QPS values obtained in Table 5 are in the range of values [0,2] which means that the performance of the Value-at-Risk ARMA-GJR-GARCH is good for use in the analyzed stock return data.

4. Conclusions

In this study, an estimation of the Value-at-Risk value has been carried out using a time series model. The data analyzed are ICBP and PGAS stock return data. The models obtained from each stock are ARMA (5,0,5)-GJR-GARCH (1,1) and ARMA (1,0,2)-GJR-GARCH (1,1) models. The results of the Value-at-Risk of each share are 0.060427 for ICBP and 0.024724 for PGAS. The maximum risk obtained by investors with an initial investment of IDR. 100,000,000.00 and invest in ICBP shares in the amount of IDR. 6,042,70,00. The risk of investors investing in PGAS shares is IDR. 2,472,400,000. From these results, it is more advisable to invest in PGAS shares, because the maximum loss borne by investors is smaller than ICBP shares. Based on QPS which is relatively small and is in the range of values [0,2], it shows that risk measurement using Value-at-Risk on the analyzed stocks has good performance. The advantages of this study are that it can estimate the value of volatility with data that has an asymmetric effect using the time series model and can determine the value of Value-at-Risk with this model. This research is useful for making decisions in investing, so it can help investors with in-stock selection.

References

Ali, G. (2013). EGARCH, GJR-GARCH, TGARCH, AVGARCH, NGARCH, IGARCH and APARCH models for pathogens at marine recreational sites. *Journal of Statistical and Econometric Methods*, 2(3), 57-73.

Bakhtiar, S. M., Syata, I., Alwi, W., Ibnuas, R., & Anugrawati, S. D. (2021, May). Estimation of average value at risk (AVaR) on sharia joint-stock index using Glosten, Jagannathan and Runkle (GJR) model. In *1st International Conference on Mathematics and Mathematics Education (ICMME 2020)* (pp. 143-149). Atlantis Press.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of econometrics*, 31(3), 307-327.

Brooks, C. (2008). RATS Handbook to accompany introductory econometrics for finance. *Cambridge Books*.

Bucevska, V. (2013). An Empirical evaluation of GARCH models in value-at-risk estimation: Evidence from the Macedonian stock exchange. *Business Systems Research: International journal of the Society for Advancing Innovation and Research in Economy*, 4(1), 49-64.

Dritsaki, C. (2017). An empirical evaluation in GARCH volatility modeling: Evidence from the Stockholm stock exchange. *Journal of Mathematical Finance*, 7(2), 366-390.

Dwipa, N. M. S. (2016). GLOSTEN JAGANNATHAN RUNKLE-GENERALIZED AUTOREGRESSIVE CONDITIONAL HETROSCEDEXASTICS (GJR-GARCH) METHODE FOR VALUE AT RISK (VaR) FORECASTING. *Proceeding of ICMSE*, 3(1), M-63.

Glosten, L. R., Jagannathan, R., & Runkle, D. E. (1993). On the relation between the expected value and the volatility of the nominal excess return on stocks. *The journal of finance*, 48(5), 1779-1801.

Lee, O. (2007). A study on Box-Cox transformed threshold GARCH (1, 1) process. *Communications for Statistical Applications and Methods*, 14(1), 141-146.

Mittnik, S., Paolella, M. S., & Rachev, S. T. (2002). Stationarity of stable power-GARCH processes. *Journal of Econometrics*, 106(1), 97-107.

Nilsson, C. (2017). Forecasting Swedish stock market volatility and value-at-risk: A comparison of EWMA and GARCH models.

Olowe, R. A. (2010). Oil price volatility, global financial crisis and the month-of-the-year effect. *International Journal of Business and Management*, 5(11), 156.

Ruppert, D., & Matteson, D. S. (2011). *Statistics and data analysis for financial engineering* (Vol. 13). New York: Springer.

Sukono, S. E., Simanjuntak, A., Santoso, A., Ghazali, P. L., & Bon, A. T. (2019, November). ARIMA-GARCH Model for Estimation of Value-at-Risk and Expected shortfall of Some Stocks in Indonesian Capital Market. In *Proceedings of the International Conference on Industrial Engineering and Operations Management Riyadh, Saudi Arabia* (pp. 327-334).

Susanti, D., Najmia, M., Lesmana, E., Napitupulu, H., Supian, S., & Putra, A. S. (2018, March). Analysis of stock investment selection based on CAPM using covariance and genetic algorithm approach. In *IOP Conference Series: Materials Science and Engineering* (Vol. 332, No. 1, p. 012046). IOP Publishing.

TamilSelvan, M., & Vali, S. M. (2016). Forecasting stock market volatility-evidence from muscat security market using garch models. *International Journal of Commerce and Finance*, 2(1), 37-53.

Tsay, R. S. (2005). *Analysis of financial time series*. John wiley & sons.