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Mean-Variance Investment Portfolio Optimization Model Without Risk-Free Assets in Jii70 Share

Shindi Adha Gusliana[1], Yasir Salih[2*]

[1] *Mathematics Undergraduate Study Program, Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran, Jatinangor, Indonesia*
Department of Mathematics, Faculty of Education, Red Sea University, SUDAN

**Corresponding author emailyasirsalih2015@gmail.com*

Abstract

In investing, investors will try to limit all the risks in managing their investments. Investor strategies to minimize investment risk are diversification by forming investment portfolios, one of which is the Mean-Variance without risk-free assets. The calculation results will show the composition of the optimum portfolio return for each stock that forms the portfolio. Optimum portfolio obtained with w^T = (0.39853, 0.25519, 0.13644, 0.09788, 0.11196) sequential weight composition for TLKM, KLBF, INCO, HRUM, and FILM stocks. The composition of this optimal portfolio return is τ 0.04 with a return of 0.00209 and a portfolio variance of 0.00015. The formation of this portfolio optimization model is expected to be additional literature in optimizing the investment portfolio with the Mean-Variance.

Keywords: Mathematics, investation

1. Introduction

Investment is a commitment to several funds made at this time to obtain profits in the future (Tandelilin 2010). Investment is a delay in current consumption to be put into productive assets for a certain period (Hartono 2017). In investing, an investor will try to limit all the risks in managing his investment. In general, risk is the degree of uncertainty that something will occur or that a goal will not be realized in a certain period. The higher the profit or return to be received, the higher the risks faced. The strategy used by investors to minimize investment risk is to diversify by forming an investment portfolio (Brigham and Houston, 2018). One of the attitudes of investors in making investment decisions tends to avoid risk. The investor's attitude can be seen when he is faced with two investments with the exact return expectations and different risks so that he will choose an investment with a lower level of risk (Fabozzi and Peterson 2003).

The normative implication drawn from the theory and the efficient market hypothesis is that investors should hold diversified portfolios. In the simplest case, an investor with no unique insight that would lead him to generate expectations that differ from market consensus is advised to hold a market portfolio combined with loans or loans. Investors would hold index funds as market proxies in a more realistic setting. Perhaps more importantly, from our perspective, even portfolio managers who are bullish or bearish on securities tend to hold slightly more or slightly less than the market weight of the security (Best and Grauer 1991). The expected return is not a definite return received. Still, it is the average result of all possible outcomes, recognizing that some outcomes have a greater chance of occurring than others from various investment scenarios (Bodie, Kane, and Marcus 2014). Moehring (2013) in his research using matrix notation to simplify the process of calculating the optimization of the Markowitz model. The Markowitz model investment portfolio selection process, in essence, allocates several funds to several assets.

The portfolio model that emphasizes the relationship between return and portfolio risk is the Markowitz model. This model is also known as the Mean-Variance. This discussion aims to show the analysis of the Mean-Variance without risk-free assets. The calculation results will show the composition of the optimum portfolio return for each stock. With the formation of this portfolio optimization model, it is hoped that it can become additional literature as an alternative for investors in optimizing investment portfolios.

2. Materials and Methods

2.1. Materials

The stock data used in forming the portfolio is as many as five stock data included in the JII 70 list of shares and traded on the capital market in Indonesia through the Indonesia Stock Exchange (IDX). The data includes data on the closing price of PT Telekomunikasi Indonesia Tbk (TLKM) shares, PT Kalbe Farma Tbk (KLBF), PT Vale Indonesia Tbk (INCO), PT Harum Energy Tbk (HRUM), and PT MD Picture Tbk (FILM). Daily stock historical data is accessed through the website www.finance.yahoo.com for one year or around 246 days, from 1 May 2021 to 30 April 2022.

2.2. Methods

Suppose given portfolio p with weight vector **w**. For an efficient portfolio, the selection is made by finding the maximum value $2\tau\mu_p - \sigma_p^2$, with the provision of $\sum_{i=1}^{N} w_i = 1$ and $\tau \geq 0$, parameter τ is called risk tolerance (De Moor et al. 2008; Ruppert 2004). In optimizing the Mean-Variance investment portfolio without risk-free assets, suppose there are N non-risk-free assets with returns r_1,…,r_N. It is assumed that the first and second moments of r_1,…,r_N exist (Basuki, Firman, and Carnia 2017). Then the transposed vector of the expected return value is expressed by:

$$\boldsymbol{\mu}^T = (\mu_1, \dots, \mu_N), \text{ with } \mu_i = E[r_i], i = 1, \dots, N \tag{1}$$

And the covariance matrix is expressed by

$$\boldsymbol{\Sigma} = (\sigma_{ij}) \text{ with } \sigma_{ij} = Cov(r_i, r_j), i, j = 1, \dots, N. \tag{2}$$

If the return portfolio r_p with the transpose weight vector $\mathbf{w}^T = (w_i, \dots, w_N)$ and terms $\sum_{i=1}^{N} w_i = 1$, then portfolio return expectations using vector notation can be expressed as:

$$\mu_p = E[r_p] = \boldsymbol{\mu}^T\mathbf{w} = \mathbf{w}^T\boldsymbol{\mu} \tag{3}$$

And the portfolio variance can be expressed as:

$$\sigma_p^2 = Var(r_p) = \mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w} \tag{4}$$

In Mean-Variance optimization, the efficient portfolio is defined as a portfolio p* is called (Mean-Variance) efficient if there is a portfolio p with $\mu_p \geq \mu_{p^*}$ and $\sigma_p^2 < \sigma_{p^*}^2$ To get an efficient portfolio, it means to solve the portfolio optimization problem as follows.

$$\text{Maximum } \{2\tau\boldsymbol{\mu}^T\mathbf{w} - \mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w}\} \tag{5}$$

$$\text{Terms: } \mathbf{e}^T\mathbf{w} = \mathbf{1}$$

With $\mathbf{e}^T = (1, \dots, N)$, $\boldsymbol{\mu}^T\mathbf{w} = \mathbf{w}^T\boldsymbol{\mu}$, and $\mathbf{w}^T\mathbf{e} = \mathbf{e}^T\mathbf{w}$.. The Lagrange function of the optimization problem where λ is the multiplier is stated as follows.

$$L(\mathbf{w}, \lambda) = (2\tau\boldsymbol{\mu}^T\mathbf{w} - \mathbf{w}^T\boldsymbol{\Sigma}\mathbf{w}) + \boldsymbol{\lambda}(\mathbf{w}^T\mathbf{e} - 1) \tag{6}$$

By using the necessary conditions, the Kuhn-Tucker theorem $\frac{\partial L}{\partial \mathbf{w}} = 0$ and $\frac{\partial L}{\partial \lambda} = 0$, obtained:

$$\frac{\partial L}{\partial \mathbf{w}} = 2\tau\boldsymbol{\mu} - 2\boldsymbol{\Sigma}\mathbf{w} + \lambda\mathbf{e} = \mathbf{0} \tag{7}$$

$$\frac{\partial L}{\partial \lambda} = \mathbf{w}^T\mathbf{e} - 1 = 0 \tag{8}$$

Equality $\frac{\partial L}{\partial \mathbf{w}}$ Which is multiplied by $\boldsymbol{\Sigma}^{-1}$ And express it in **w**, then multiply the result $\mathbf{e}^T$, so that the following solution is obtained:

$$\mathbf{w} = \frac{1}{\mathbf{e}^T\boldsymbol{\Sigma}^{-1}\mathbf{e}}\boldsymbol{\Sigma}^{-1}\mathbf{e} + \tau\left\{\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu} - \frac{\mathbf{e}^T\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}}{\mathbf{e}^T\boldsymbol{\Sigma}^{-1}\mathbf{e}}\boldsymbol{\Sigma}^{-1}\mathbf{e}\right\} \; ; \; \tau \geq 0 \tag{9}$$

When τ=0, it produces a minimum variance portfolio with weights:

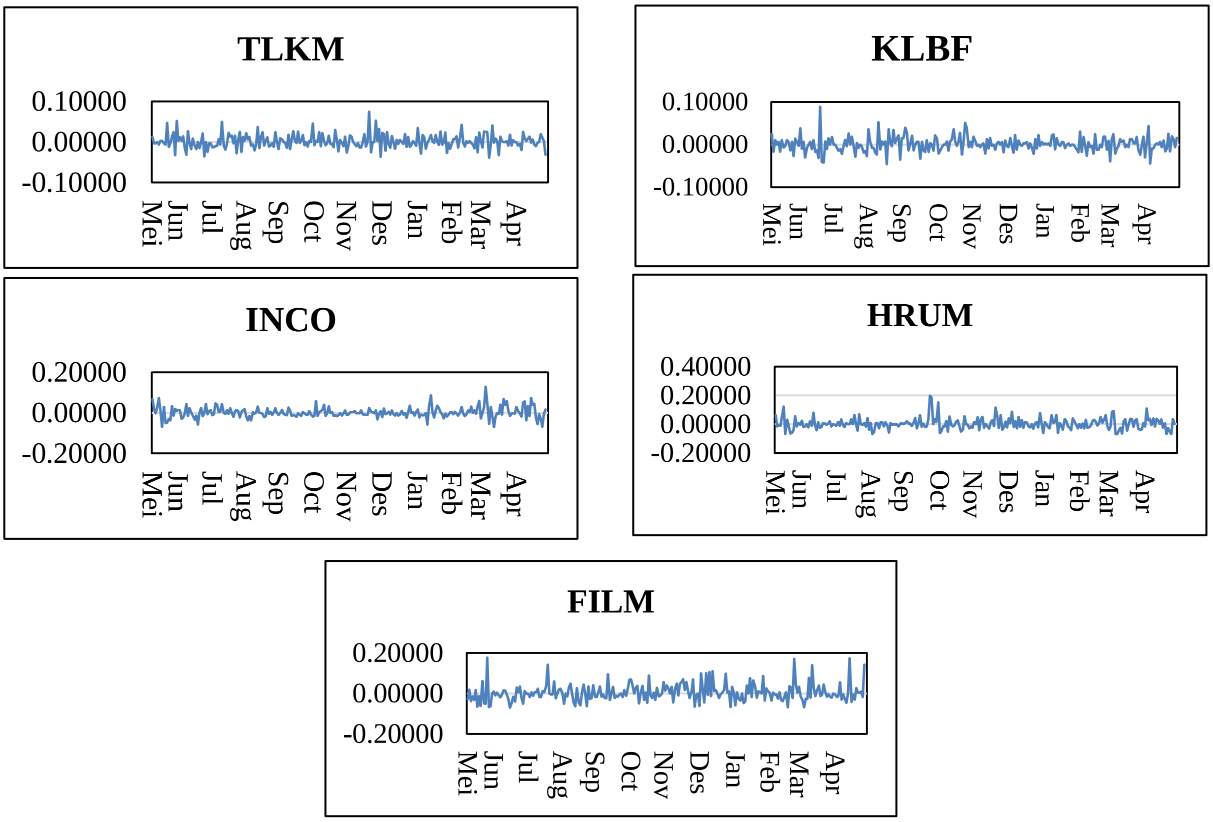
$$\mathbf{w}^{Min} = \frac{1}{\mathbf{e}^T\boldsymbol{\Sigma}^{-1}\mathbf{e}}\boldsymbol{\Sigma}^{-1}\mathbf{e} \tag{10}$$

3. Results and Discussion

3.1 Stock Data Analysis

3.1.1 Stock Return

The formation of return charts for five stocks, namely TLKM, KLBF, INCO, HRUM, and FILM, including the

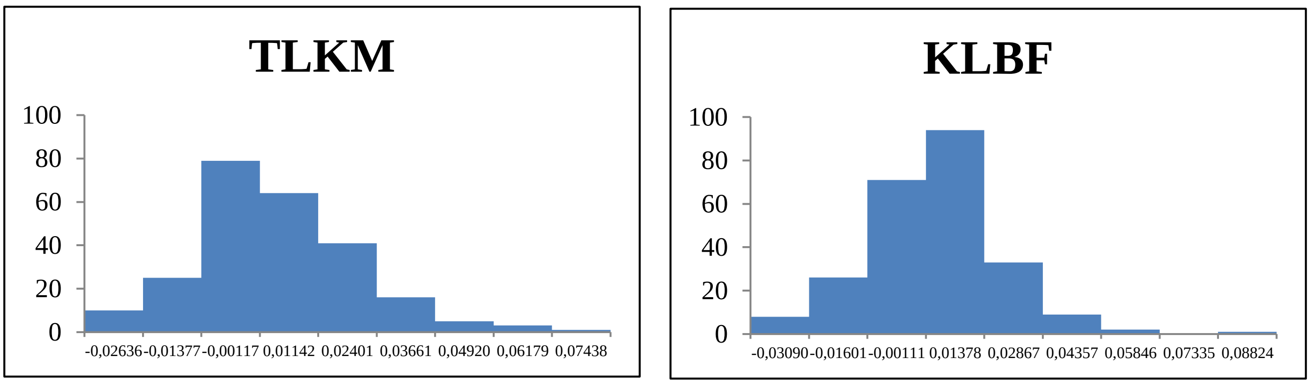


following:

Figure 1: Stock Return Chart

3.1.2 Descriptive Statistics

In this section, the distribution model is identified by making a histogram of stock returns of the five stocks, which are as follows.



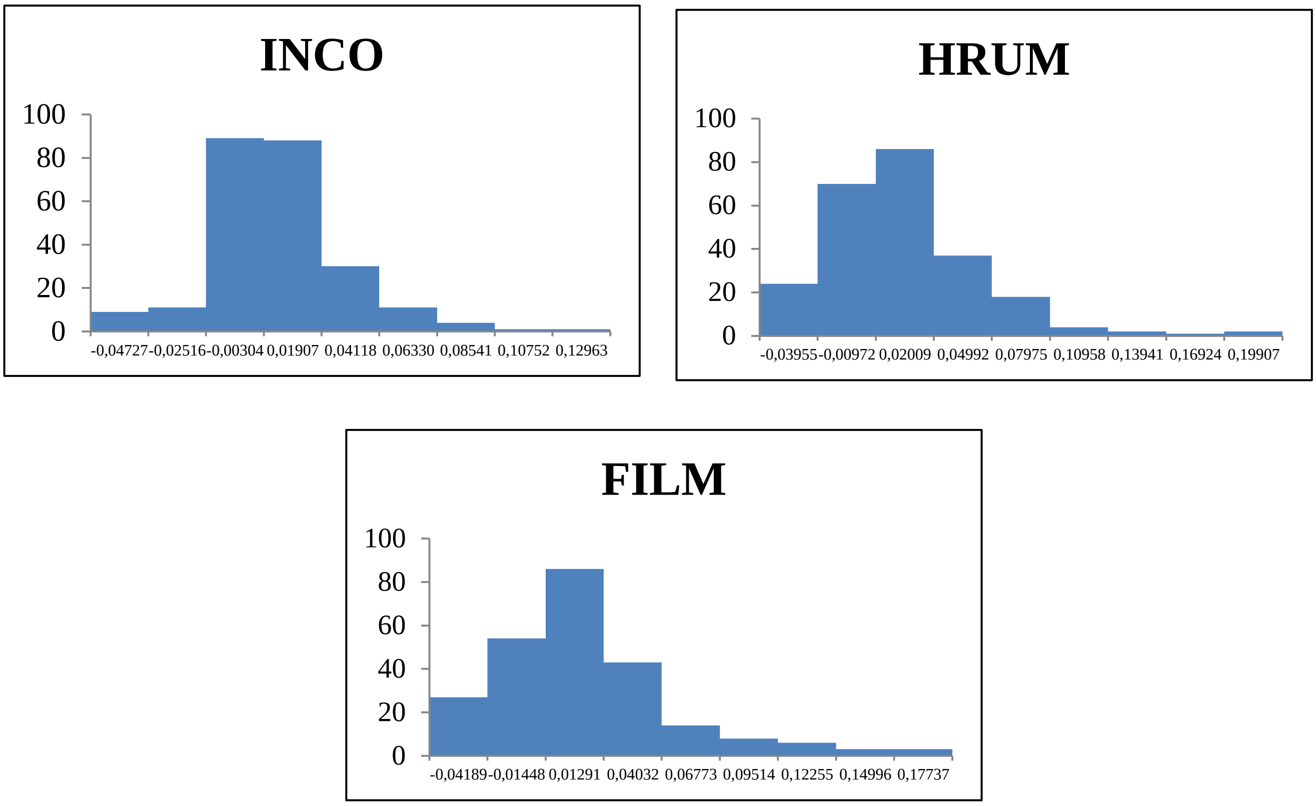


Figure 2: Stock Return Histogram

Based on the stock return histogram in Figure 2, it can be seen that the histogram is generally shaped like a bell, so it can be assumed that the return distribution follows the normal distribution. The results of the estimated distribution, expectations, and variance of returns from the five stocks, along with the ratio between expectations and variances of returns, can be seen in Table 1.

Table 1: Distribution Estimation, Expectation, and Stock Return Variance

Share Name	Distribution Estimator	Expectation/ μ means	Variance σ^2	Ratio μ/σ^2
TLKM	Normal	0.00169	0.00029	5.81798
KLBF	Normal	0.00070	0.00030	2.32650
INCO	Normal	0.00225	0.00067	3.36170
HRUM	Normal	0.00421	0.00161	2.61636
FILM	Normal	0.00466	0.00187	2.49459

Of the five stocks, then the estimated covariance value between shares is determined. With the help of excel software, the results are shown in Table 2

Table 2: Stock Covariance Estimation

	TLKM	**KLBF**	**INCO**	**HRUM**
TLKM	0.00029	0.00003	0.00006	0.00004
KLBF	0.00003	0.00030	0.00002	0.00006
INCO	0.00006	0.00002	0.00067	0.00031
HRUM	0.00004	0.00006	0.00031	0.00161
FILM	0.00005	0.00002	0.00009	0.00010

3.2 Formation of Mean-Variance Investment Portfolio Optimization Without Risk-Free Assets

From the average value estimator in Table 1, μ_i (i= 1, ..., 5) form the mean transpose vector $\boldsymbol{\mu}^T$ = (0.000169, 0.00070, 0.00225, 0.00421, 0.00466). Then a unit transpose vector is formed e^T = (1 1 1 1 1). Next, estimate the variance value $\sigma_i^2, (i = 1, \ldots, 5)$ as well as the results of the calculation of the covariance estimator between stock returns in Table 2. the covariance matrix Σ is formed as follows:

$$\Sigma = \begin{bmatrix} 0.00029 & 0.00003 & 0.00006 & 0.00004 & 0.00005 \\ 0.00003 & 0.00030 & 0.00002 & 0.00006 & 0.00002 \\ 0.00006 & 0.00002 & 0.00067 & 0.00031 & 0.00009 \\ 0.00004 & 0.00006 & 0.00031 & 0.00161 & 0.00010 \\ 0.00005 & 0.00002 & 0.00009 & 0.00010 & 0.00187 \end{bmatrix}$$

With the help of excel software, the inverse matrix can be determined Σ^{-1}, as follows.

$$\Sigma^{-1} = \begin{bmatrix} 3{,}577.98 & -279.59 & -301.03 & -6.92 & -72.64 \\ -279.59 & 3{,}339.51 & -18.60 & -109.25 & -26.40 \\ -301.03 & -18.60 & 1{,}665.43 & -305.37 & -58.36 \\ -6.92 & -109.25 & -305.37 & 686.32 & -21.32 \\ -72.64 & -26.40 & -58.36 & -21.32 & 540.38 \end{bmatrix}$$

Next, the matrix inverse Σ^{-1} used to calculate the composition of the efficient portfolio weight is carried out based on the Mean-Variance portfolio optimization model.

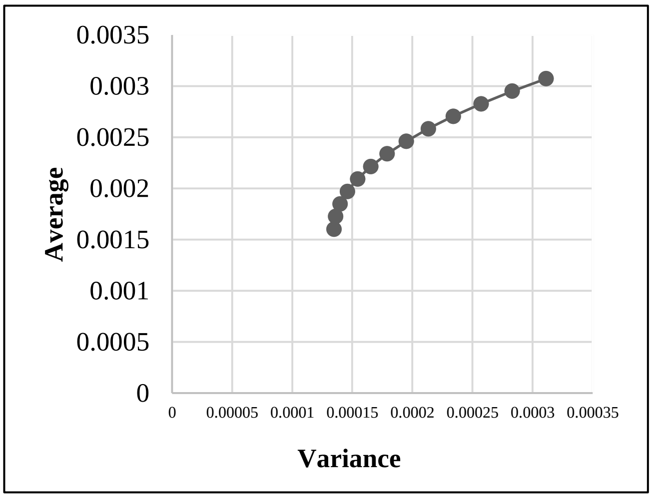
3.3 Mean-Variance Investment Portfolio Optimization Process Without Risk-Free Assets

On the problem of optimizing the Mean-Variance portfolio without risk-free assets, using vectors $\boldsymbol{\mu}^T$ and $\mathbf{e}^T$ with matrix Σ^{-1}, then the weight vector $\mathbf{w}$ is calculated. Risk tolerance τ with conditions $\tau \geq 0$ in investment portfolio optimization, simulated by taking several values that meet the requirements $\mathbf{e}^T\mathbf{w} = \mathbf{1}$. Taking the risk tolerance value is stopped if a value for a risk tolerance value produces a weight w_i, ($i = 1, ..., 5$) which is not a positive actual number and satisfies $\mathbf{e}^T\mathbf{w} = \mathbf{1}$. The results of taking risk tolerance values and calculating efficient portfolio weights are shown in Table 3.

Table 3: Mean-Variance Investment Portfolio Optimization Process Without Risk-Free Assets on JJI70 shares

τ	TLKM	KLBF	INCO	HRUM	FILM	e^Tw	μ_p	σ_p^2	μ_p/σ_p^2
0	0.39373	0.39209	0.13252	0.03285	0.04880	1	0.00160	0.00013	11.87549
0.01	0.39493	0.35787	0.13350	0.04911	0.06459	1	0.00172	0.00014	12.66816
0.02	0.39613	0.32364	0.13448	0.06537	0.08038	1	0.00185	0.00014	13.21114
0.03	0.39733	0.28942	0.13546	0.08162	0.09617	1	0.00197	0.00015	13.49597
0.04	**0.39853**	**0.25519**	**0.13644**	**0.09788**	**0.11196**	**1**	**0.00209**	**0.00015**	**13.53978**
0.05	0.39973	0.22097	0.13742	0.11414	0.12775	1	0.00221	0.00017	13.37808
0.06	0.40092	0.18674	0.13840	0.13039	0.14354	1	0.00234	0.00018	13.05546
0.07	0.40212	0.15252	0.13937	0.14665	0.15933	1	0.00246	0.00019	12.61745
0.08	0.40332	0.11829	0.14035	0.16291	0.17512	1	0.00258	0.00021	12.10497
0.09	0.40452	0.08407	0.14133	0.17916	0.19091	1	0.00270	0.00023	11.55163
0.10	0.40572	0.04984	0.14231	0.19542	0.20670	1	0.00283	0.00026	10.98315
0.11	0.40692	0.01562	0.14329	0.21168	0.22249	1	0.00295	0.00028	10.41801
0.12	0.40812	-0.01860	0.14427	0.22794	0.23828	1	0.00307	0.00031	9.86872

A series of efficient portfolios are on the efficient frontier, an efficient surface where the return portfolios are commensurate with the risks.



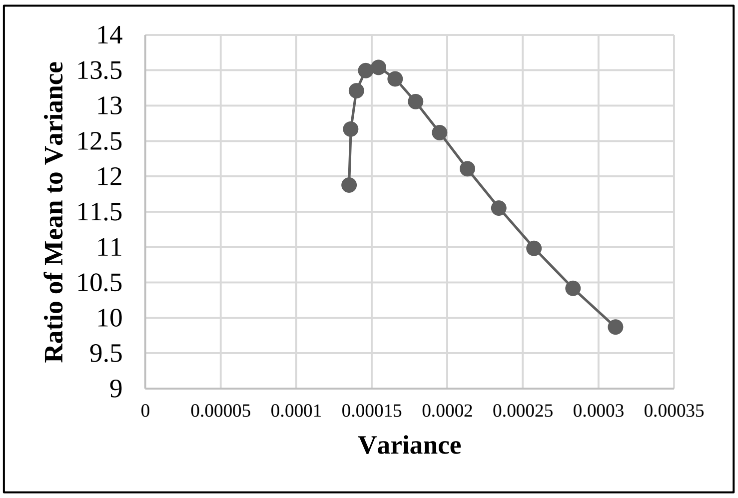


Figure 2: Efficient Frontier and Mean-Variance Portfolio Ratio Without Risk-Free Assets

3.4 Discussion

The risk tolerance for the Mean-Variance model without risk-free assets ranges from $0 \le \tau \le 0.11$, The optimum portfolio is obtained, which is composed of five stocks, namely a portfolio with a weighted composition $\mathbf{w}^{\mathbf{T}}$ = (0.39853, 0.25519, 0.13644, 0.09788, 0.11196) sequentially for TLKM, KLBF, INCO, HRUM, and FILM shares. The composition of this optimal portfolio return on $\tau = 0.04$ by producing an average return value of 0.00209 and a portfolio variance of 0.00015.

4. Conclusion

To determine the optimum investment portfolio weight allocation on assets without risk-free, the Mean-Variance investment portfolio optimization model can be used as Markowitz's basic model. Expansion of the Mean-Variance is done by entering risk-free assets in JII70 shares. With this expansion, a formula can be derived to determine the optimum portfolio weight allocation. From the investment portfolio optimization model, an optimum portfolio is obtained, composed of five stocks included in the JII70 stock list, namely the composition of this optimal portfolio return on τ = 0.04 by producing an average return value of 0.00209 and a portfolio variance of 0.00015.

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