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Selection of the Best B-Spline Regression Model for Estimating Bitcoin Price Increases Based on Order and Optimal Knot Point

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Abstract

The cryptographic virtual currency, bitcoin, is considered the main originator of cryptocurrencies that emerged due United States financial crisis in 2008. The idea was sparked by Nakamoto by introducing an alternative currency system that really refers to the strength of supply and demand. Based on INDODAX data, the bitcoin exchange rate during October 2020 to February 2021 is a condition of a large increase in a short time with a percentage increase of 450%. The increase in bitcoin prices can be modelled using the b-spline nonparametric regression method based on order and optimal knot points based on the smallest Generalized Cross Validation value. The resulting b-spline 4 degree and the number of knots points 5 as the best model with each bases described recursively.

Keywords: Bitcoin, cryptocurrency, nonparametric, b-spline, generalized cross validation.

1. Introduction

Utilization of technology in the era of the digital revolution is a major requirement in financial activities. The rapid development of technology and science, especially in the field of the internet, has increased economic growth in the fields of trade, finance and investment. Internet technology is able to have an impact on transaction activities using bitcoin as a means of payment (Bhiantara, 2018).

The cryptographic virtual currency called bitcoin is the first decentralized peer-to-peer blockchain network that is completely controlled by users without any intermediaries (Sovbetov, 2018). Blockchain is a series of blocks arranged sequentially where each block has a list of transaction information. This cryptocurrency idea is in accordance with the requirements for a legal medium of exchange, namely mutually agreed upon, not easily damaged and unique (Wong, 2014).

Bitcoin was created by Satoshi Nakamoto who appeared because the United States experienced the Great Recession in 2008. The principle of bitcoin is to create a system of decentralized authority transactions without intermediaries, so that the concept of digital signatures can be used as a means of verification in every transaction (Nakamoto, 2008). The idea created by Nakamoto introduced an alternative money system that leads to demand and supply forces, namely price changes occur because of the large number of goods offered (Prayogo, 2018).

Bitcoin buying and selling services were first provided by New Liberty Standard on October 5, 2009, where the initial price was 1 United States Dollar equivalent to 1,309.03 BTC or about eight hundredths of a cent per bitcoin. This price is obtained from the use of electricity costs on computers to produce bitcoins. The first bitcoin exchange was carried out by New Liberty Standard by buying 5.05 BTC from Sirius for $ 5.02 using PayPal on October 12, 2009. As of February 18, 2021 the bitcoin exchange rate was at $ 50,856 or equivalent to Rp. 737,412,000.00 while the previous 4 months were on the 15th October 2020 the bitcoin exchange rate was at $ 11,521.7241 or equivalent to Rp. 167,065,000.00, this was one of the biggest increases in the history of the formation of bitcoin with an increase percentage approaching 450%.

Research regarding entrance exam scores on the GPA of UK Visual Communication Design Students. Petra Surabaya by Budiantara et al (2006) using b-spline regression explains that linear, quadratic, and cubic b-spline models have the same MSE value, so overall the best model is selected based on minimum MSA, GCV and coefficient of determination. Research conducted by Dung & Tjahjowidodo (2017) regarding methods for optimizing b-spline curve knot points resulted in a new method for calculating optimal knots b-spline regression based on local b-spline installation techniques in cases of non-uniform knots.

2. B-Spline Nonparmetric Regression

Nonparametric regression is a method used to determine the form of relationship between an independent variable y and the dependent variable x with a curve model whose regression function is unknown (Qiao, 2015). Nonparametric regression is written as follows (Suwasono et al, 2010):

$$y_i = f(x_i) + \varepsilon_i \,, \tag{1}$$

nonparametric regression model with observations (x_i, y_i) is:

$$y_i = \mu(x_i) + \varepsilon_i, \qquad i = 1,2,\dots,n, \tag{2}$$

with ε_i is the residual and $\mu(x_i)$ value of the function μ unknown at that point $x_1 \dots x_n$ assumed $a_0 \le x_1 \dots x_n \le a_1$ when a_0 is the minimum value x and a_1 is the maximum value x.

Regression models $y_i = f(x_i) + \varepsilon_i$, with ε_i is an error and $f(x_i)$ is the regression function. If the regression curve f approximated by the b-spline function then f written as:

$$f(x) = \sum_{j=1}^{m+k} b_j N_{j-m,m}(x)\,, \tag{3}$$

with $N_{j-m,m}$ namely the basis of the b-spline of the order $(j-m)$.

The steps form a base b-spline of order m for the knot point $a_0 < \xi_1 \dots \xi_k < a_1$ is to determine as many additional knots $2m$ with $\xi_{-(m-1)} < \cdots < \xi_{-1} < \xi_0 < \cdots < \xi_{k+m}$ when $\xi_{-(m-1)} = \cdots = \xi_0 = a_0$ and $\xi_{k+1} = \cdots = \xi_{k+m} = a_1$. Value $b_1, b_2, \dots, b_{m+k}$ is an estimator.

The bases of b-spline functions are categorized by order m as follows:

a. Base b-spline with $m = 2$ gives a linear function which can be written as follows:

$$N_{i,2}(x) = \frac{x-\xi_i}{\xi_{i+1}-\xi_i} N_{i,1}(x) + \frac{\xi_{i+2}-x}{\xi_{i+2}-\xi_{i+1}} N_{i+1,1}(x),$$

b. The b-spline basis with $m = 3$ gives a quadratic function which can be written as follows:

$$N_{i,3}(x) = \frac{x-\xi_i}{\xi_{i+2}-\xi_i} N_{i,2}(x) + \frac{\xi_{i+3}-x}{\xi_{i+3}-\xi_{i+1}} N_{i+1,2}(x),$$

c. The b-spline basis with $m = 4$ gives a cubic function which can be written as follows:

$$N_{i,4}(x) = \frac{x-\xi_i}{\xi_{i+3}-\xi_i} N_{i,3}(x) + \frac{\xi_{i+4}-x}{\xi_{i+4}-\xi_{i+1}} N_{i+3}(x),$$

The estimated coefficient λ on a b-spline basis is defined in matrix form as follows:

$$\boldsymbol{N}(\lambda) = \left(N_{j,m}(x_i)\right) \quad i = 1,2,\dots,n; j = -(m-1),\dots,k\,,$$

or it can be written as follows:

$$\boldsymbol{N}(\lambda) = \begin{bmatrix} N_{-(m-1),m}(x_1) & N_{-(m-2),m}(x_1) & \cdots & N_{k,m}(x_1) \\ N_{-(m-1),m}(x_2) & N_{-(m-2),m}(x_2) & \cdots & N_{k,m}(x_2) \\ \vdots & \vdots & \ddots & \vdots \\ N_{-(m-1),m}(x_n) & N_{-(m-2),m}(x_n) & \cdots & N_{k,m}(x_n) \end{bmatrix}, \tag{4}$$

where $\boldsymbol{N}(\lambda)$ is a matrix of size$n \times (m+k)$ (Botella & Shariff, 2003).

In order to obtain the best b-spline regression model, you need to pay attention to the placement of the knot points $k_1, k_2, \dots, k_n$ the optimal one. The formation of knot points makes the curve focused on predetermined knot points so that placing knot points must be in accordance with the distribution of the formed data.

Selection of the best b-spline regression using generalized cross validation (GCV) values, the minimum GCV value defines the optimal knot point. The optimal knot point shows that the resulting regression model is the best model. The GCV value can be written in the following equation (Qiao, 2015):

$$GCV(k_1, k_2, \dots, k_n) = \frac{MSE(k_1,k_2,\dots,k_n)}{\left(\frac{1}{n} trace[I - A(k_1,k_2,\dots,k_n)]\right)^2}\,. \tag{5}$$

The b-spline model in nonparametric regression of order m with k knot points in equation (3) can be written as:

$$y_i = b_1 N_{1-m,m}(x_i) + b_2 N_{2-m,m}(x_i) + \cdots + b_{(m+k)} N_{k,m}(x_i) + \varepsilon_i. \tag{6}$$

If the b-spline model is presented in matrix form, we get:

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} N_{1-m,m}(x_1) & N_{2-m,m}(x_1) & \cdots & N_{k,m}(x_1) \\ N_{1-m,m}(x_2) & N_{1-m,m}(x_2) & \cdots & N_{k,m}(x_2) \\ \vdots & \vdots & \vdots & \vdots \\ N_{1-m,m}(x_i) & N_{1-m,m}(x_i) & \cdots & N_{k,m}(x_n) \end{pmatrix} \begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_{(m+k)} \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{pmatrix},$$

which is then written as:

$$\boldsymbol{y} = \boldsymbol{N}\,\boldsymbol{b} + \boldsymbol{\varepsilon},$$

The regression curve μ is approximated by a b-spline function of order m with k knot points for $\lambda = \{\xi_1, \dots, \xi_k\}$ presented in the form:

$$\mu_\lambda(x_i) = \sum_{j=1}^{m+k} b_{\lambda j}\, N_{j-m,m}(x_i),$$

with a b-spline model for $\lambda = \{\xi_1, \dots, \xi_k\}$ is:

$$y_i = \sum_{j=1}^{m+k} b_{\lambda j}\, N_{j-m,m}(x_i) + \varepsilon_i. \quad (7)$$

The regression model in equation (7) can be written as:

$$y_i = b_{\lambda 1} N_{1-m,m}(x_i) + b_{\lambda 2} N_{2-m,m}(x_i) + \cdots + b_{\lambda(m+k)} N_{k,m}(x_i) + \varepsilon_i, \quad i = 1,2, \dots n$$

The above equation if written in matrix form will get:

$$\begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{pmatrix} = \begin{pmatrix} N_{1-m,m}(x_1) & N_{2-m,m}(x_1) & \cdots & N_{k,m}(x_1) \\ N_{1-m,m}(x_2) & N_{1-m,m}(x_2) & \cdots & N_{k,m}(x_2) \\ \vdots & \vdots & \vdots & \vdots \\ N_{1-m,m}(x_i) & N_{1-m,m}(x_i) & \cdots & N_{k,m}(x_n) \end{pmatrix} \begin{pmatrix} b_{\lambda 1} \\ b_{\lambda 2} \\ \vdots \\ b_{\lambda(m+k)} \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_n \end{pmatrix},$$

which can be written as:

$$\boldsymbol{y} = \boldsymbol{N_\lambda}\, \boldsymbol{b_\lambda} + \boldsymbol{\varepsilon}. \quad (8)$$

Parameter estimation $\boldsymbol{b_\lambda} = (b_{\lambda 1} \quad b_{\lambda 2} \quad \cdots \quad b_{\lambda(m+k)})^T$ Parameter estimates were obtained using the least squares spline method. Estimator $\boldsymbol{b_\lambda}$ obtained by simplifying the sum of squared errors or the Residual Sum of Squares (RSS) which is defined as follows:

$$\begin{aligned} \boldsymbol{RSS} &= \boldsymbol{\varepsilon^T \varepsilon} \\ &= (\boldsymbol{y} - \boldsymbol{N_\lambda b_\lambda})^{\boldsymbol{T}} (\boldsymbol{y} - \boldsymbol{N_\lambda b_\lambda}) \\ &= \left(\boldsymbol{y^T} - \boldsymbol{b_\lambda}^{\boldsymbol{T}} \boldsymbol{N_\lambda}^{\boldsymbol{T}}\right)(\boldsymbol{y} - \boldsymbol{N_\lambda b_\lambda}) \\ &= \boldsymbol{y^T y} - \boldsymbol{b_\lambda}^{\boldsymbol{T}} \boldsymbol{N_\lambda}^{\boldsymbol{T}} \boldsymbol{y} - \boldsymbol{y^T N_\lambda b_\lambda} + \boldsymbol{b_\lambda}^{\boldsymbol{T}} \boldsymbol{N_\lambda}^{\boldsymbol{T}} \boldsymbol{N_\lambda b_\lambda} \\ &= \boldsymbol{y^T y} - \boldsymbol{b_\lambda}^{\boldsymbol{T}} \boldsymbol{N_\lambda}^{\boldsymbol{T}} \boldsymbol{y} - \left(\boldsymbol{b_\lambda}^{\boldsymbol{T}} \boldsymbol{N_\lambda}^{\boldsymbol{T}} \boldsymbol{y}\right)^{\boldsymbol{T}} + \boldsymbol{b_\lambda}^{\boldsymbol{T}} \boldsymbol{N_\lambda}^{\boldsymbol{T}} \boldsymbol{N_\lambda b_\lambda} \\ \boldsymbol{RSS} &= \boldsymbol{y^T y} - \boldsymbol{2 b_\lambda}^{\boldsymbol{T}} \boldsymbol{N_\lambda}^{\boldsymbol{T}} \boldsymbol{y} + \boldsymbol{b_\lambda}^{\boldsymbol{T}} \boldsymbol{N_\lambda}^{\boldsymbol{T}} \boldsymbol{N_\lambda b_\lambda}. \end{aligned} \quad (9)$$

Then equation (9) is derived from $\boldsymbol{b_\lambda}$ to obtain:

$$\frac{\partial RSS}{\partial \boldsymbol{b_\lambda}} = 2\boldsymbol{N_\lambda}^{\boldsymbol{T}} \boldsymbol{y} + \boldsymbol{2 N_\lambda}^{\boldsymbol{T}} \boldsymbol{N_\lambda b_\lambda},$$

RSS results derived against $\boldsymbol{b_\lambda}$ equate to zero to:

$$-2\boldsymbol{N_\lambda}^{\boldsymbol{T}} \boldsymbol{y} + 2\boldsymbol{N_\lambda}^{\boldsymbol{T}} \boldsymbol{N_\lambda b_\lambda} = 0, \quad (10)$$

Equation (10) has the following solution:

$$\begin{aligned} \boldsymbol{N_\lambda}^{\boldsymbol{T}} \boldsymbol{N_\lambda} \boldsymbol{\hat{b}_\lambda} &= \boldsymbol{N_\lambda}^{\boldsymbol{T}} \boldsymbol{y} \\ \boldsymbol{\hat{b}_\lambda} &= \left(\boldsymbol{N_\lambda}^{\boldsymbol{T}} \boldsymbol{N_\lambda}\right)^{\boldsymbol{-1}} \boldsymbol{N_\lambda}^{\boldsymbol{T}} \boldsymbol{y}, \end{aligned}$$

with $\boldsymbol{\hat{b}_\lambda} = \left(\hat{b}_{\lambda 1} \quad \hat{b}_{\lambda 2} \quad \cdots \quad \hat{b}_{\lambda(m+k)}\right)^T$.

Regression curve estimator $\hat{\mu}_\lambda = (\hat{\mu}_{\lambda 1} \quad \hat{\mu}_{\lambda 2} \quad \cdots \quad \hat{\mu}_{\lambda(n)})^T$ is given by:

$$\begin{aligned} \hat{\mu}_\lambda &= \boldsymbol{N_\lambda \hat{b}_\lambda} \\ &= \boldsymbol{N_\lambda} \left(\left(\boldsymbol{N_\lambda}^{\boldsymbol{T}} \boldsymbol{N_\lambda}\right)^{\boldsymbol{-1}} \boldsymbol{N_\lambda}^{\boldsymbol{T}} \boldsymbol{y} \right) \\ &= \boldsymbol{N_\lambda} \left(\boldsymbol{N_\lambda}^{\boldsymbol{T}} \boldsymbol{N_\lambda}\right)^{\boldsymbol{-1}} \boldsymbol{N_\lambda}^{\boldsymbol{T}} \boldsymbol{y} \\ \hat{\mu}_\lambda &= \boldsymbol{S_\lambda y}, \end{aligned}$$

with matrix $\boldsymbol{S_\lambda} = \boldsymbol{N_\lambda}\left(\boldsymbol{N_\lambda}^{\boldsymbol{T}} \boldsymbol{N_\lambda}\right)^{\boldsymbol{-1}} \boldsymbol{N_\lambda}^{\boldsymbol{T}}$ symmetrical and positive definite (Eubank, 1999).

The estimator for the regression curve can be written $\hat{\mu}_\lambda(t) = \sum_{j=1}^{m+k} \hat{b}_{\lambda j}\, N_{j-m,m}(x)$ with $\hat{b}_{\lambda j}$ obtained from $\boldsymbol{\hat{b}_\lambda} = \left(\hat{b}_{\lambda 1} \quad \hat{b}_{\lambda 2} \quad \cdots \quad \hat{b}_{\lambda(m+k)}\right)^T$. So the estimation for the ¬b-spline function is as follows:

$$\hat{y} = \sum_{j=1}^{m+k} \hat{b}_{\lambda j}\, N_{j-m,m}(x), \quad (11)$$

Equation (11) can also be written as follows:

$$\hat{y} = \hat{b}_{\lambda 1} N_{1-m,m}(x) + \hat{b}_{\lambda 2} N_{2-m,m}(x) + \cdots + \hat{b}_{\lambda(m+k)} N_{k,m}(x). \quad (12)$$

3. Research Methodology

The data used in this research is data obtained from the INDODAX website (Indodax.com, 2021). This data includes the increase in bitcoin (BTC) with a time span from January 2020 to February 2021. The increase in the price of bitcoin is analyzed per week, to make calculations easier using R studio, the data is converted into the form $n \times 10^{-8}$. The stages in this research are:

a. Look for GCV values for linear, quadratic and cubic b-spline regression models, which are then calculated sequentially from 1 knot point to 5 knot points.
b. b-spline regression modeling at every order and knot point. The best model is chosen based on the model with the smallest generalized cross validation value.
c. Find estimated parameter values using the least squares spline method. The parameter estimates are obtained by simplifying the sum of squared errors.
d. Perform value substitution into the best b-spline regression model. The model produces b-spline bases which are then described recursively in the solution.

4. Result and Discussion

Selection of The Best B-Spline Regression Model for Estimating Bitcoin Price Increases

Using R studio, the results of the calculation of generalized cross validation for each order and knot point of the b-spline regression are presented in Table 1.

Table 1: Optimal GCV value based on model estimation

B-Spline	Number of Knots	Point Knots	GCV
Linear (m=2)	1	1.09669	0.11592460
	2	6.84669, 7.34669	0.09426707
	3	5.44669, 6.64669, 6.84669	0.06979342
	4	4.24669, 4.34669, 6.84669, 7.34669	0.05051962
	5	4.24669, 4.34669, 4.54669, 6.84669, 7.14669	0.03709815
Quadratic (m=3)	1	1.12669	0.11995150
	2	6.84669, 7.24669	0.08517687
	3	5.44669, 6.04669, 6.14669	0.06435183
	4	5.24669, 5.34669, 5.84669, 6.04669	0.05434509
	5	4.54669, 4.74669, 5.24669, 5.44669, 6.64669	0.04369458
Cubic (m=4)	1	4.69669	0.12082050
	2	6.84669, 7.34669	0.07316846
	3	5.44669, 5.54669, 5.64669	0.06248605
	4	5.24669, 5.34669, 5.44669, 5.64669	0.05519905
	5	**4.44669, 4.54669, 5.14669, 5.34669, 5.74669**	**0.03679738**

Determination of the best b-spline regression model is obtained from the smallest GCV value of all orders and knot points. Based on Table 1, the best estimator is found in the cubic b-spline model with a GCV value of 0.03679738 at knot points one, two, three, four and five of 4.44669, 4.54669, 5.14669, 5.34669 and 5.74669.

Next, the model that will be formed based on the best estimated value in cubic b-spline regression using five knot points is as follows:

$$\hat{y} = \hat{b}_{\lambda 1}N_{-3,4}(x) + \hat{b}_{\lambda 2}N_{-2,4}(x) + \hat{b}_{\lambda 3}N_{-1,4}(x) + \hat{b}_{\lambda 4}N_{0,4}(x) + \hat{b}_{\lambda 5}N_{1,4}(x) + \hat{b}_{\lambda 6}N_{2,4}(x) + \hat{b}_{\lambda 7}N_{3,4}(x) + \hat{b}_{\lambda 8}N_{4,4}(x) + \hat{b}_{\lambda 9}N_{5,4}(x), \quad (13)$$

The $\boldsymbol{b_\lambda}$ estimator is obtained using the least squares spline method. The estimator $\boldsymbol{b_\lambda}$ for m orders and k knot points is $\boldsymbol{b_\lambda} = (b_{\lambda 1} \quad b_{\lambda 2} \quad \cdots \quad b_{\lambda(m+k)})^T$ which is obtained by simplifying the sum of squared errors. The results of the calculation of the $\boldsymbol{b_\lambda}$ estimator are presented in Table 2.

Table 2: Estimator value $\boldsymbol{b_\lambda}$

Estimator	Value	Estimator	Value
$b_{\lambda 1}$	0.75523	$b_{\lambda 6}$	21.38250
$b_{\lambda 2}$	2.43103	$b_{\lambda 7}$	-73.17607
$b_{\lambda 3}$	1.92647	$b_{\lambda 8}$	36.41619
$b_{\lambda 4}$	5.67681	$b_{\lambda 9}$	6.80423
$b_{\lambda 5}$	-6.77883		

The estimator value $\boldsymbol{b_\lambda}$ is substituted into equation (13) to become:

$$\hat{y} = 0.75523N_{-3,4}(x) + 2.43103N_{-2,4}(x) + 1.92647N_{-1,4}(x) + 5.67681N_{0,4}(x) - 6.77883N_{1,4}(x) + 21.38250N_{2,4}(x) - 73.17607N_{3,4}(x) + 36.41619N_{4,4}(x) + 6.80423N_{5,4}(x). \quad (14)$$

After getting the estimator value $\boldsymbol{b_\lambda}$, Next, the estimated value of the parameter estimator is calculated using the b-spline function bases which are categorized based on the number of orders. Next define as many additional knots as possible $2m$ with $a_0 < \xi_1 < \cdots < \xi_k < a_1$ when $\xi_{-(m-1)} = \cdots = \xi_0 = a_0$ and $\xi_{k+1} = \cdots = \xi_{k+m} = a_1$ where is value a_0 obtained from the minimum $x = 0.83669$ and value a_1 obtained from the maximum $x = 7.37412$. Substitute additional knots by replacing values $m = 4$ and $k = 5$ so that additional knot points can be written as:

$$\xi_{-3} = \cdots = \xi_0 = 0.83669, \xi_1 = 4.44669, \xi_2 = 4.54669, \xi_3 = 5.14559,$$
$$\xi_4 = 5.34669, \xi_5 = 5.74669, \xi_6 = \cdots = \xi_9 = 7.37412.$$

The basis of the b-spline function in equation (14) is defined recursively as follows:

$$N_{-3,4}(x) = \frac{x - \xi_{-3}}{\xi_0 - \xi_{-3}} N_{-3,3}(x) + \frac{\xi_1 - x}{\xi_1 - \xi_{-2}} N_{-2,3}(x)$$
$$N_{-3,4}(x) = 0 + \frac{4.44669 - x}{3.61} N_{-2,3}(x), \quad (15)$$

deciphering the bases in equation (16)

$$N_{-2,3}(x) = \frac{x - \xi_{-2}}{\xi_0 - \xi_{-2}} N_{-2,2}(x) + \frac{\xi_1 - x}{\xi_1 - \xi_{-1}} N_{-1,2}(x)$$
$$N_{-2,3}(x) = 0 + \frac{4.44669 - x}{3.61} N_{-1,2}(x), \quad (16)$$

deciphering the bases in equation (16)

$$N_{-1,2}(x) = \frac{x - \xi_{-1}}{\xi_0 - \xi_{-1}} N_{-1,1}(x) + \frac{\xi_1 - x}{\xi_1 - \xi_0} N_{0,1}(x)$$
$$N_{-1,2}(x) = 0 + \frac{4.44669 - x}{3.61} N_{0,1}(x). \quad (17)$$

Substitute equation (17) into equation (16)

$$N_{-2,3}(x) = \left(\frac{4.44669 - x}{3.61}\right)\left(\frac{4.44669 - x}{3.61} N_{0,1}(x)\right)$$
$$N_{-2,3}(x) = \frac{(4.44669 - x)^2}{(3.61)^2} N_{0,1}(x), \quad (18)$$

Substitute equation (18) into equation (15)

$$N_{-3,4}(x) = \left(\frac{4.44669 - x}{3.61}\right)\left(\frac{(4.44669 - x)^2}{(3.61)^2} N_{0,1}(x)\right)$$
$$N_{-3,4}(x) = \frac{(4.44669 - x)^3}{(3.61)^3} N_{0,1}(x), \quad (19)$$

so that equation (20) can then be written as follows:

$$N_{-3,4}(x) \begin{cases} \frac{(4.44669 - x)^3}{(3.61)^3} & , 0.83669 \le x \le 4.44669 \\ 0 & , \text{other} \end{cases}$$

The solution for the b-spline bases is then used with the same steps as the steps above.

5. Conclusion

Modeling the increase in the bitcoin price from January 2020 to February 2021 using b-spline regression produces b-spline models at every order and knot point. The best model is selected based on the parameter estimation results with the smallest GCV reference. The smallest GCV value is obtained at the number 0.03679738 which is generated in the 4th order b-spline regression model and the number of knot points is 5 with each base being translated recursively.

References

Bhiantara, I. B. P. (2018). Cryptocurrency blockchain technology in the era of digital revolution. *Universitas Pendidikan Ganesha,* 173–177.

Botella, O., & Shariff, K. (2003). B-spline methods in fluid dynamics. *International Journal of Computational Fluid Dynamics, 17*(2), 133–149.

Budiantara, I. N., Suryadi, F., Otok, B. W., & Guritno, S. (2006). B-spline and Mars modeling on entrance exam scores on the GPA of UK Visual Communication Design Department students. Petra Surabaya. Journal of Industrial Engineering, Universitas Kristen Petra, 8(1), 1-13.

Dung, V. T., & Tjahjowidodo, T. (2017). A direct method to solve optimal knots of Bspline curves: An application for non-uniform b-spline curves fitting. *Plos One, 12*(3), 1–24.

Eubank, R. L. (1999). *Nonparametric Regression and Spline Smoothing.* Florida: CRC Press.

Indodax. (2021). Bitcoin Market. https://indodax.com/market/BTCIDR. Accessed 22 Februari 2021 Pk. 20.30 WIB.

Nakamoto, S. (2008). Re: Bitcoin P2P e-cash Paper. https://users.encs.concordia.ca/. Accessed 25 Maret 2021 Pk. 20.00 WIB.

Prayogo, G. (2018). Bitcoin, regulation and the importance of national legal reform. *Asian Journal of Law and Jurisprudence, 1*(1), 1-9.

Sovbetov, Y. (2018). Factors influencing cryptocurrency prices: evidence from bitcoin, ethereum, dash, litcoin, and monero. *Journal of Economics and Financial Analysis, 2*(2), 1–27.

Suwasono, B., Widjaja, S., Zubaydi, A., Yuliadi, Z., & Budiantara, I. N. (2010). Development of technological parameters for cubic spline prediction. *Makara, 14*(2), 121–127.

Wong, W. (2014). *Bitcoin : A Practical Guide to Understanding, Mining, and Earning Bitcoin*. Bogor: Indraprasta Media.

Qiao, B., Chen, X., Xue, X., Luo, X., & Liu, R. (2015). The application of cubic B-spline collocation method in impact force identification. *Mechanical Systems and Signal Processing, 64,* 413-427.