



Pricing of Fisheries Microinsurance Premiums using the Poisson-Exponential Aggregate Distribution Approach

Dila Nur Fadhillah^{1*}, Raynita Shahla²

^{1,2}*Undergrad Program in Mathematics, Faculty of Mathematics and Sciences, Universitas Padjadjaran, Sumedang, Jawa Barat, Indonesia*

**Corresponding author email: dila20001@mail.unpad.ac.id*

Abstract

Engaged in pond aquaculture is currently an attractive choice amid the high demand for fish in the market. Entrepreneurial opportunities in the pond fish farming sector are increasingly open, although the risk of crop failure remains, both due to weather factors and livestock processes. Crop failure can have a significant financial impact on pond fishery farmers. Therefore, there is a need for special insurance to protect against financial losses due to risks that can occur, namely Micro Fisheries Insurance. Microinsurance is a type of insurance product specifically designed for people with low income levels, offers features and administration that is simple, easily accessible, has an economical price, and a fast compensation settlement process. The focus of this study is to calculate premium prices by applying an aggregate risk model approach. The data used are the number of events and the magnitude of losses due to crop failure in shrimp pond cultivation in Pandeglang Regency in the period January 1, 2019-January 1, 2021. Data on the number of events follow the Poisson distribution, while data on the magnitude of losses follow the Exponential distribution. Next, it uses the *Maximum Likelihood Estimation* (MLE) method to calculate parameter estimation. The average and variance of aggregate risk is used to determine the size of the premium. The premium selection results in this study amounted to IDR 42,005,600. The amount of the premium reflects the collective premium resulting from the calculation based on the standard deviation principle.

Keywords: Fisheries Microinsurance, premium determination, aggregate risk model, poisson distribution, exponential distribution.

1. Introduction

Entrepreneurial opportunities in the aquaculture sector are increasingly wide open, presenting lucrative profit prospects for farmers. However, in aquaculture, ponds cannot be separated from risks, especially in the context of the risk of crop failure. Based on this, this study focuses on managing financial risks in aquaculture ponds through the application of Micro Fisheries Insurance (Bueno, 2008; Secretan, 2007).

Microinsurance is considered as a potential solution to protect pond farmers from financial impacts that may arise due to the risk of crop failure. Therefore, this study focuses on calculating premium prices by applying an aggregate risk model approach (Tsanakas & Desli, 2005; Cairns et al., 2006). By collecting and analyzing data from shrimp pond farming in Pandeglang District, this study aims to contribute in determining microinsurance premiums in accordance with local risk characteristics (Röttger, 2015).

The results of this research will be the basis for determining the amount of premium that reflects the level of risk faced by pond fishery farmers in the region. This research is expected to provide a deeper understanding of the risks and benefits of micro-fisheries insurance in the context of aquaculture, as well as provide a foundation for wider implementation at the local and regional levels.

2. Theoretical Foundation

Septiana (2022) conducted a study entitled "Determination of Insurance Premiums for Losses Due to Crop Failure in Pond Cultivation with the Principle of Expectation Value". The results of the study showed the premium value of fishermen's losses due to not going to sea based on the principle of expectation of IDR 25,893,046.00 and with the principle of standard deviation of IDR 23,539,132.00. Meanwhile, research by Kusumadewi (2022) entitled

"Determination of Fishermen Microinsurance Premium Prices Using the Aggregate Risk Model Approach in Cirebon Regency" also provides an understanding of microinsurance premium pricing for fishermen. The results of this study show the premium value of fishermen's losses due to not going to sea based on the principle of expectation of Rp. 162,547,612.00 and with the principle of standard deviation of IDR 153,861,958.00. Both studies provide an important foundation related to risk assessment and premium setting. Based on this, this study will apply the principle of expectation to the aggregate risk model to determine the premium price of microfisheries insurance, especially in the context of shrimp pond cultivation in Pandeglang Regency.

3. Research Objects and Methods

3.1. Object

The object used in this study is data on crop failure experienced by fishery farmers sourced from Septiana's research (2022). From these data, microinsurance premium will be calculated using an aggregate risk model approach. The data used is loss data from the capture of shrimp ponds that failed to harvest in Pandeglang Regency. The tools used are Microsoft Excel and Easyfit software.

3.2. Method

3.2.1. Discrete Random Variable Distribution

The distribution used is the Poisson distribution which is to determine the frequency of events. A Poisson-distributed random variable with parameters if has the following probability function: $N\lambda > 0$

$$P(N = x) = e^{-\lambda} \frac{\lambda^x}{x!}. \quad (1)$$

The moment generating function of equation (1) is as follows:

$$M_N(t) = \sum_{x=0}^{\infty} e^{tx} e^{-\lambda} \frac{\lambda^x}{x!} = \exp\{\lambda(e^t - 1)\}. \quad (2)$$

The probability generating function of equation (2) is as follows:

$$P(r) = \sum_{x=0}^{\infty} r^x e^{-\lambda} \frac{\lambda^x}{x!} = \exp\{\lambda(r - 1)\}. \quad (3)$$

Based on equation (3) the expectations and variances of the Poisson distribution are obtained as follows:

$$E(N) = \lambda \quad (4)$$

$$Var(N) = \lambda. \quad (5)$$

To estimate the parameters of the Poisson distribution, suppose is a random sample of a population that the Poisson distribution with the parameter λ . N_1, N_2, \dots, N_n

$$L(\lambda) = \prod_{i=1}^n f(x_i, \lambda) = \frac{e^{-n\lambda} \lambda^{\sum x_i}}{\prod_{i=1}^n x_i!} \quad (6)$$

$$\ln L(\lambda) = -n\lambda \ln e + \sum x_i \ln \lambda - \ln \prod_{i=1}^n x_i!. \quad (7)$$

The estimation of the Poisson distribution parameter is obtained by maximizing the function in equation (7) as follows: $\ln L(\lambda)$

$$\frac{d(\ln L(\lambda))}{d\lambda} = 0, \quad (8)$$

until it is obtained

$$\hat{\lambda}_{MLE} = \frac{\sum x_i}{n} \quad (9)$$

3.2.2. Continuous Distribution of Random Variables

The distribution used is the Exponential distribution. A random variable X is Exponentially distributed with the parameter if it has the probability density function as follows: μ

$$f(x) = \frac{1}{\mu} e^{-\frac{x}{\mu}}. \quad (10)$$

Based on equation (10), the distribution function for is as follows: $x > 0$

$$F(x) = -e^{-\frac{x}{\mu}}. \quad (11)$$

The functions of the moment generator of are as follows: X

$$M_X(t) = \frac{1}{1 - \mu t}, t < \mu, \quad (12)$$

So that the moment of the exponential distribution is as follows: n

$$E(X^n) = n! \mu^n. \quad (13)$$

Based on equation (10) we get the expectation and variance of the Exponential distribution:

$$E(X) = \mu \quad (14)$$

$$Var(X) = \mu^2 \quad (15)$$

To estimate the parameters of the Exponential distribution, suppose is a random sample of a population that is Exponentially distributed with the parameter μ . X_1, X_2, \dots, X_n

$$L(\mu) = \prod_{i=1}^n f(x_i, \mu) = \left(\frac{1}{\mu}\right)^n e^{-\frac{1}{\mu} \sum_{i=1}^n x_i} \quad (16)$$

$$\ln L(\mu) = -n \ln \mu - \frac{1}{\mu} \sum_{i=1}^n x_i. \quad (17)$$

The estimation of the Exponential distribution parameter is obtained by maximizing the function in equation (17) as follows:

$$\frac{d(\ln L(\mu))}{d\mu} = 0. \quad (18)$$

until it is obtained

$$\hat{\mu}_{MLE} = \bar{x}. \quad (19)$$

3.2.3. Distribution Conformity Test

The distribution suitability tests used are *Kolmogorov Smirnov* and *Chi-Square*.

$$D = \max\{|F_k(x) - F_0(x)|\} \quad (20)$$

H_0 : data follows a specific distribution

H_1 : the data does not follow a specific distribution

The assumption of such distribution decisions is if $H_0 D > D_{tabel}$

$$\chi^2 = \sum_{i=1}^G \frac{(O_i - E_i)^2}{E_i}, \quad (21)$$

with

χ^2 : computed chi-squared parameter

G : number of subgroups

O_i : the number of observation values in subgroup I

E_i : the number of theoretical values in subgroup I

3.2.4. Collective Risk Model

The collective risk model is formulated as follows:

$$S = X_1 + X_2 + \dots + X_{N(t)} = \sum_{i=1}^{N(t)} X_i, \quad (22)$$

where is a random variable that expresses N the number of events and is a random variable that expresses the magnitude of losses. From equation (22) the expectations and variances of collective risk are obtained as follows: $X_1, X_2, \dots, X_{N(t)}$

$$E(S) = E(N)E(X) \quad (23)$$

$$Var(S) = E(N)Var(X) + Var(N)(E(X))^2 \quad (24)$$

3.2.5. Premium Calculation Model

The premium calculation model with the principle of expected value is formulated as follows:

$$p(t) = (1 + \alpha)E(S(t)), \quad (25)$$

with $0 < \alpha < 1$

While the premium calculation model with the standard deviation principle is formulated as follows:

$$p(t) = E(S(t)) + \alpha \sqrt{Var(S(t))}, \quad (26)$$

with $0 < \alpha < 1$

4. Results and Discussion

The data used in this study was obtained through a survey conducted by Septiana (2022). The data includes crop losses that occurred to shrimp farmers in Pandeglang Regency in the period from January 2019 to January 2021. Loss data can be classified into two categories, namely data on the number of events and data on the amount of losses.

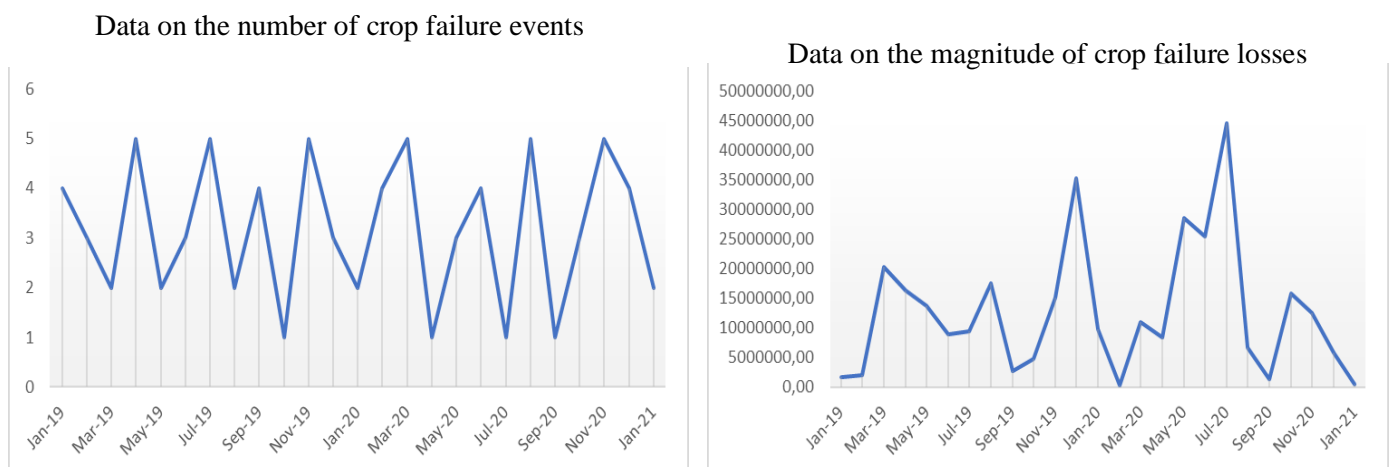


Figure 1: (a) Data on the Number of Crop Failure Events, (b) Data on the Magnitude of Crop Failure Losses

In determining the amount of premium that must be paid by shrimp farmers as a preparation step to face potential losses due to crop failure, it can be done by assessing expectations and variances from collective loss risks. Furthermore, the estimator is applied to formulate the premium size with the premium model of expectation and standard deviation principles. In this study, the process of processing and calculating data using the help of Microsoft Excel and EasyFit software.

4.1 Multi-Event Model

The process of identifying the distribution model is carried out by creating a frequency distribution histogram that records the number of events. From the results of the processing, it is obtained as Figure 2.

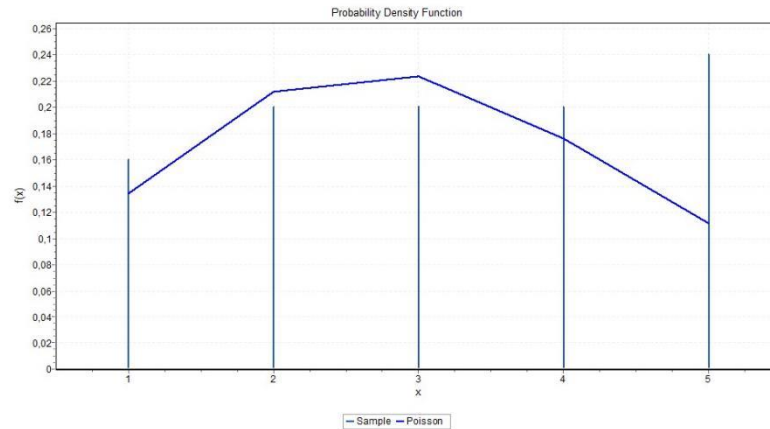


Figure 2: Histogram and Data Curve of Multiple Events

Based on Figure 2, it can be assumed that data on the number of crop failure events following the Poisson distribution will then be tested for distribution suitability using the *Kolmogorov-Smirnov* test at the 95% significance level described in Table 1.

Table 1: *Kolmogorov-Smirnov* Match Test Results Poisson Distribution on Multi-Event Data

Many Occurrences	Test Statistics (D)	0.25143
	D_{tabel}	0.26404
	Result	$D < D_{tabel}$ or $0.25143 < 0.26404$
	Conclusion	H_0 received, Poisson distributed event multiplicity data

Next, the parameter assessment of the number of events will be carried out using the *Maximum Likelihood Estimation* (MLE) method of the Poisson distribution which refers to equation (8).

$$\begin{aligned}
 \frac{d(\ln L(\lambda))}{d\lambda} &= 0 \\
 \frac{d(-25\lambda \ln e + \sum_{i=1}^{25} x_i \ln \lambda - \ln \prod_{i=1}^{25} x_i!)}{d\lambda} &= 0 \\
 -25 + \frac{\sum_{i=1}^{25} x_i}{\hat{\lambda}} &= 0 \\
 \hat{\lambda}_{MLE} &= \frac{\sum_{i=1}^{25} x_i}{25} \\
 \lambda &= 3,16.
 \end{aligned} \tag{27}$$

The next step is the calculation of collective risk using the expectation value based on equation (4) and variance based on equation (5) of the Poisson distribution with the parameter $\lambda = 3,16$. The opportunity density function of the Poisson distribution is as follows:

$$P(N = x) = \frac{e^{-3.16}(3.16)^x}{x!}, \tag{18}$$

so that the expected value $E(N) = \lambda = 3.16$ and the variance value are obtained $Var(N) = \lambda = 3.16$.

4.2 Model the magnitude of losses

The process of identifying the distribution model is carried out by making a frequency distribution histogram that records the magnitude of losses. From the results of the processing, it is obtained as Figure 3.

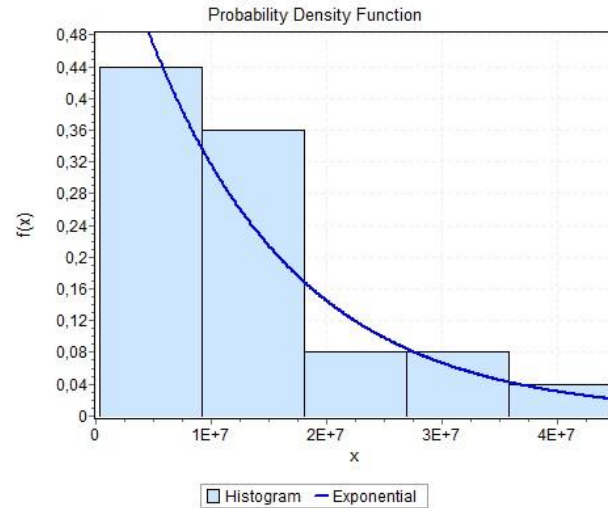


Figure 3: Histogram and Data Curve of Loss Magnitude

Based on Figure 3, it can be assumed that the data on the magnitude of crop failure losses follows the Exponential distribution which will then be tested for distribution suitability using the *Chi-Square* test at the 95% significance level described in Table 2.

Table 2: Exponential Distribution *Chi-Square* Match Test Results on Loss Magnitude Data

Many Occurrences	Test Statistics (χ^2)	0.26409
	χ_{cr}^2	2.5018
	Result	$\chi^2 < \chi_{cr}^2$ or $0.26409 < 2.5018$
	Conclusion	H_0 received, Exponentially distributed event count data

Next, we will assess the parameters of the number of events using the *Maximum Likelihood Estimation* (MLE) method of the Exponential distribution which refers to equation (18).

$$\begin{aligned}
 \frac{d(\ln L(\mu))}{d\mu} &= 0 \\
 -\frac{25}{\hat{\mu}} + \frac{1}{\hat{\mu}^2} \sum_{i=1}^{25} x_i &= 0 \\
 \hat{\mu} &= \frac{1}{25} \sum_{i=1}^{25} x_i \\
 \hat{\mu}_{MLE} &= \bar{x} \\
 \mu &= 12,784,367
 \end{aligned} \tag{29}$$

The next step is the calculation of collective risk using the expectation value based on equation (14) and variance based on equation (15) of the Exponential distribution with parameter $\mu = 12,784,367$. The probability density function of the Exponential distribution is as follows:

$$F(x; \mu) = \int_0^x \mu e^{-\mu x}, \tag{30}$$

so obtained $E(N) = \mu = 12,784,367$ and $Var(N) = \mu^2 = (12,784,367)^2 = 163,440,039,590,689$

4.3 Risk Calculation and Premium Value

The next step is to calculate the overall level of risk. The estimated value of expectations and variance of collective risk can be calculated using equations (23) and (24) respectively.

$$E(S) = (3.16)(12,784,367) = 40,398,600 \quad (31)$$

$$Var(S) = (3.16)(163,440,039,590,689) + (3.16)(12,784,367)^2 = 1,032,941,050,213,154 \quad (32)$$

When determining the amount of premium, the main factors taken into account involve how much loss the shrimp farmer has suffered and the number of failures during the harvest period. Applying the principle of expected value according to equation (25), the calculation of the premium value is as follows:

$$p(t) = (1 + 0.1) \cdot 40,398,600 = 44,438,460. \quad (33)$$

Furthermore, by applying the principle of standard deviation based on equation (26), the premium calculation is as follows:

$$p(t) = 40,398,600 + 0.05\sqrt{1,032,941,050,213,154} = 42,005,600 \quad (34)$$

Thus, a premium amount is obtained that can serve as a storage of funds to overcome losses that may be experienced by shrimp farmers during the harvest period. This is achieved through the application of the expected value principle and the standard deviation principle obtained from the overall loss risk estimation as illustrated in Table 3.

Table 3: Calculation of Premium Premium for Shrimp Farm Losses Experiencing Crop Failure

Calculation Method	Premium Amount/year
Expectation Principle	IDR 44,438,460
Standard Deviation Principle	IDR 42,005,600

The results of the premium calculation are used as a reference for insurance companies in determining the amount of individual microinsurance premium prices that must be paid by shrimp farmers. The premium value of IDR 42,005,600 was chosen because the value is lower than other calculations so that it can result in a lower individual premium price also by the insurance company.

5. Conclusion

Based on data analysis and discussion of research results, several conclusions can be drawn. First, in the period from January 1, 2019 to January 1, 2021, it shows that the number of crop failure events is in the range of 1 to 5 times, while data related to losses cover the range of IDR 345,724 to IDR 44,671,851. Second, data from the number of crop failure events is data that follows the Poisson distribution with an estimated parameter λ of $\lambda = 3.16$. The estimation of the expectations and variances of the Poisson distribution is and the $E(N) = 3.16$ value of its variance $Var(N) = 3.16$. Meanwhile, the data on the magnitude of crop failure losses follows the Exponential distribution with an estimated parameter of μ $\mu = 12,784,367$. The estimated expectations of the variance of this exponential distribution are $E(N) = 12,784,367$ and $Var(N) = 163,440,039,590,689$. Finally, based on the principle of expectation value with *loading factor*, the $\alpha = 0.05$ premium for shrimp pond losses due to crop failure has a value of IDR 44,438,460, while based on the standard deviation principle has a value of IDR 42,005,600.

References

- Bueno, P. B. (2008). Social risks in aquaculture. *Understanding and applying risk analysis in aquaculture*, 209.
- Cairns, A. J., Blake, D., & Dowd, K. (2006). A two-factor model for stochastic mortality with parameter uncertainty: theory and calibration. *Journal of Risk and Insurance*, 73(4), 687-718.
- Kusumadewi, R (2022) *Pricing of Fishermen's Microinsurance Premiums Using an Aggregate Risk Model Approach in Cirebon Regency*. Unpublished thesis. Sumedang: Faculty of Mathematics and Natural Sciences Unpad Jatinangor.
- Röttger, D. (2015). *Agricultural finance for smallholder farmers: Rethinking traditional microfinance risk and cost management approaches* (Vol. 11). Columbia University Press.
- Secretan, P. A. (2007). *Guidelines to meet insurance and other risk management needs in developing aquaculture in Asia* (No. 496). Food & Agriculture Org.

- Septiana, F. I (2022) *Determination of Loss Insurance Premiums Due to Crop Failure in Pond Cultivation with the Expectation Value Principle*. Unpublished thesis. Sumedang: Faculty of Mathematics and Natural Sciences Unpad Jatinangor.
- Tsanakas, A., & Desli, E. (2005). Measurement and pricing of risk in insurance markets. *Risk Analysis: An International Journal*, 25(6), 1653-1668.