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Analysis of Premium Reserves Value in Endowment Life Insurance Using Fackler Method

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Abstract

Every insurance company undoubtedly seeks to gain profit, and one of the avenues for profit comes from premium reserves, which represent the amount of money held by the insurance company during the coverage period. Various methods, including the Frackler method employed in this study, are utilized to calculate premium reserves. In the calculation process, the author requires the premium value and the sum insured paid, inputting them into the Frackler method formula. The research findings indicate that the age of an insured individual at the time of enrolling in endowment life insurance influences the premium value. Consequently, it can be concluded that as the age of the insured increases, the premium reserve required for the policy tends to be higher.

Keywords: Life Insurance, endowment life insurance, premium reserve, fackler method.

1. Introduction

The realm of life insurance and its associated financial intricacies continually evolves, reflecting advancements in methodologies designed to ensure precision and reliability. One such methodology that has garnered attention is the Frackler method, particularly in the determination of premium reserves for endowment life insurance. As the insurance landscape becomes more nuanced, actuarial sciences continually adapt to meet the challenges posed by a dynamic financial environment (Shen, 2020).

This paper delves into the nuanced world of endowment life insurance and the application of the Frackler method in establishing premium reserves. The Frackler method, known for its analytical rigor, provides a systematic approach to evaluating risks, thereby influencing the calculation of reserves for endowment policies. Through an exploration of this method, we aim to unravel its complexities, shedding light on how it shapes financial decisions within the life insurance sector (Ananda, 2023).

As we embark on this exploration, our objective is to provide insights into the Frackler method's application, its implications for policyholders and insurers, and its role in maintaining the stability and reliability of endowment life insurance products. By understanding the nuances of this methodology, stakeholders can make informed decisions that align with the evolving landscape of the insurance industry.

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2. Literature Review

2.1. Insurance

Insurance is a willingness to allocate certain and small losses as a substitute for uncertain and large losses. From the above formulation, it can be concluded that people are willing to pay for small losses in the present so that they can face potential large losses that may occur in the future (Gupta, 2011).

2.1.1. Life Insurance

Life insurance is a form of insurance designed to cover individuals against unexpected financial losses resulting from premature death or living too long. Here, it is illustrated that in life insurance, the risks faced are the risk of death and the risk of living too long. This, of course, will involve various aspects, particularly if the risks inherent in an individual are not insured with a life insurance company (Gupta, 2011).

2.1.1.1. Endowment Life Insurance

In endowment life insurance, payments are made either when an individual pass away within a specified period or when they remain alive. The premiums for endowment life insurance are higher compared to term life insurance. Endowment life insurance encompasses elements of both term life insurance and pure endowment. It can be utilized, for example, to cover future educational expenses for children. The difference from term life insurance lies in the fact that if the contract expires, the insured sum will not be lost and can be received back. The duration of the contract depends on the agreement specified by the involved parties (Gupta, 2011).

2.2. Interest Rate

The amount of payment made by the borrower to the lender, usually including the interest, is referred to as the interest rate. The magnitude of the interest typically depends on the principal amount, the duration, and the interest rate (Neumeyer, 2005). There are two methods of calculating interest, namely

2.2.1. Compound Interest

Compound interest is a calculation of interest where the principal amount of the next investment period is the previous principal amount plus the interest earned (Futami, 1993). The formula for the total principal with its interest is as follows

$$S = P(1+i)^n \tag{1}$$

In compound interest, there is a known interest rate function denoted by v and can be written as follows

$$v = \frac{1}{1+i} \tag{2}$$

As a result, equation (1) can be written as

$$P = \frac{S}{(1+i)^n} = v^n S$$

v can also be defined as the present value of a payment of 1 made 1 year later.

Furthermore, a discount rate function is defined and can be written as follows

$$d = 1 - v = \frac{i}{1+i}$$

Because v is the present value of a payment of 1 made 1 year later, if the payment is made 1 year earlier, the amount of interest lost is d.

2.3. Mortality Table

The life table, commonly known as the mortality table, is a crucial tool in life insurance, as it determines the premium rates for policyholders. This table contains information about the likelihood of an individual's death based on age. Therefore, the most important column in the mortality table is the ${}_nq_x$ column, where ${}_nq_x$ represents the probability of a person dying at age x before reaching age $x+n$ (Mishra, 2011). The components of the mortality table used in this study are D_x, N_x, C_x, and M_x.

2.4. Commutation Symbols

In this section, several commutation symbols are explained. Commutation symbols are utilized to calculate single premiums, annual premiums, reserve values, and other crucial calculations within insurance. The purpose behind the creation of these symbols is to simplify calculations. The prerequisite for using commutation symbols is having knowledge of a mortality table and the interest rate. The following are the symbols used in insurance calculations,

$$D_x = v^x l_x \tag{3}$$
$$N_x = D_x + D_{x+1} + D_{x+2} + \cdots + D_\omega \tag{4}$$
$$S_x = N_x + N_{x+1} + N_{x+2} + \cdots + S_\omega \tag{5}$$
$$C_x = v^{x+1} d_x \tag{6}$$
$$M_x = C_x + C_{x+1} + C_{x+2} + \cdots + C_\omega \tag{7}$$
$$R_x = M_x + M_{x+1} + M_{x+2} + \cdots + M_\omega \tag{8}$$

2.5. Annuity

An annuity is a fixed payment made at regular intervals and for a specific duration (Futami, 1993). There are two commonly known types of annuities: certain annuities and life annuities. In terms of the timing of payments, annuities can also be categorized into two types: immediate annuities, where payments are made at the beginning of the year, and deferred annuities, where payments are made at the end of the year.

2.5.1. Life Annuity Certain

A life annuity certain is an annuity whose payments are made for a specific period (Futami, 1993). Let l_x represent the number of individuals alive and aged x with an annuity of 1 for a duration of n; for the deferred life annuity, it can be denoted as $a_{x:\overline{n}|}$, and its equation is given by

$$a_{x:\overline{n}|} = \frac{v l_{x+1} + v^2 l_{x+2} + v^3 l_{x+3} + \cdots + v^n l_{x+n}}{l_x}$$

multiply the numerator and denominator by v^x, then we obtain

$$a_{x:\overline{n}|} = \frac{v^{x+1} l_{x+1} + v^{x+2} l_{x+2} + v^{x+3} l_{x+3} + \cdots + v^{x+n} l_{x+n}}{v^x l_x}$$

based on equation (3) then

$$a_{x:\overline{n}|} = \frac{D_{x+1} + D_{x+2} + D_{x+3} + \cdots + D_{x+n}}{D_x}$$

based on equation (4) then

$$a_{x:\overline{n}|} = \frac{N_{x+1} + N_{x+n+1}}{D_x}$$

for the immediate life annuity with an annuity of 1 and denoted as $\ddot{a}_{x:\overline{n}|}$, its equation is

$$\ddot{a}_{x:\overline{n}|} = 1 + \frac{v^{x+1} l_{x+1} + v^{x+2} l_{x+2} + v^{x+3} l_{x+3} + \cdots + v^{x+n-1} l_{x+n-1}}{v^x l_x}$$
$$\ddot{a}_{x:\overline{n}|} = 1 + \frac{D_{x+1} + D_{x+2} + D_{x+3} + \cdots + D_{x+n-1}}{D_x}$$
$$\ddot{a}_{x:\overline{n}|} = \frac{D_{x+1} + D_{x+2} + D_{x+3} + \cdots + D_{x+n-1}}{D_x}$$
$$\ddot{a}_{x:\overline{n}|} = \frac{N_x - N_{x+n}}{D_x} \tag{9}$$

2.6. Single Premium

Single premium is a premium payment where the premium is paid when the insurance contract is approved, and thereafter, no further payments are made.

2.6.1. Single Premium of Endowment Life Insurance

The single premium of endowment life insurance for an individual aged x with a duration of n years and a sum assured of 1 is denoted by $A_{x:\overline{n}|}$

$$A_{x:\overline{n}|} = A^{1}_{x:\overline{n}|} + A^{\;\;1}_{x:\overline{n}|}$$

based on the single premium of whole-life insurance and term-life insurance formula, then

$$A_{x:\overline{n}|} = \frac{M_x + M_{x+n}}{D_x} + \frac{D_{x+n}}{D_x}$$

$$A_{x:\overline{n}|} = \frac{M_x - M_{x+n} + D_{x+n}}{D_x} \tag{10}$$

2.7. Annual Premium

Annual premium is a premium whose payment is made at the beginning of each year. If the amount of the premium paid each year is the same, then this annual premium is called a standard premium. There are three types of annual premiums based on the types of life insurance products, namely, annual premium for whole life insurance, annual premium for term life insurance, and annual premium for endowment life insurance.

2.7.1. Annual Premium of Endowment Life Insurance

The annual premium for endowment life insurance for an individual aged x with a sum assured of 1 and a term of n years is denoted by $P_{x:\overline{n}|}$.

$$P_{x:\overline{n}|} = \frac{A_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}} \tag{11}$$

According to equations (9) and (10), then

$$P_{x:\overline{n}|} = \frac{M_x \dfrac{M_x - M_{x+n} + D_{x+n}}{D_x}}{\dfrac{N_x - N_{x+n}}{D_x}}$$

$$P_{x:\overline{n}|} = \frac{M_x - M_{x+n} + D_{x+n}}{N_x - N_{x+n}} \tag{12}$$

3. Materials and Methods

3.1. Materials

The research subject utilized in this study is the simulation data from five policyholders of endowment life insurance with a premium amount of IDR1,000,000 for a twenty-year term, and the calculation will focus on the reserve value for the last five years are given in Table 1.

Table 1. Five Policyholders of Endowment Life Insurance

No.	Name	Age	Insurance Coverage Period	Premium
1.	AC	26	20 years	IDR1,000,000.00
2.	SA	31	20 years	IDR1,000,000.00
3.	IF	32	20 years	IDR1,000,000.00
4.	MR	22	20 years	IDR1,000,000.00
5.	KI	28	20 years	IDR1,000,000.00

In this research, data in the form of the Indonesian Mortality Table (TMI) 1999 for males, issued by the Indonesian Life Insurance Association (AAJI), was also utilized. The author employed Microsoft Excel 2019 software to facilitate the data analysis process.

3.2. Methods

The Fackler method is a derivative of the general formula for retrospective reserves. In the retrospective reserve, it can be expressed that the reserve for each year is as follows:

$$_{1}V = \frac{(l_x P(1+i) - d_x)}{l_{x+1}}$$

$$_{2}V = \frac{(l_{x+1} \cdot {}_{1}V + l_{x+1}P)(1+i) - d_{x+1}}{l_{x+2}}$$

$$_{3}V = \frac{(l_{x+2} \cdot {}_{2}V + l_{x+2}P)(1+i) - d_{x+2}}{l_{x+3}}$$

$$\vdots$$

$$_{t}V = \frac{(l_{x+t-1} \cdot {}_{t-1}V + l_{x+t-1}P)(1+i) - d_{x+t-1}}{l_{x+t}} \quad (13)$$

Because in this study, the insurance used is endowment insurance, P can be replaced with $P_{x:\overline{n|}}$, and equation (12) can be written.

$$_{t}V_{x;\overline{n|}} = \frac{(l_{x+t-1} \cdot {}_{t-1}V_{x;\overline{n|}} + l_{x+t-1}P_{x;\overline{n|}})(1+i) - d_{x+t-1}}{l_{x+t}} \quad (14)$$

The assumption of the Fackler method explains that the determined final reserve value is the reserve at the end of the following year. In other words, the sought-after reserve value is the reserve for year t + 1. In finding the reserve at the end of year t + 1 using equation (14), the equation obtained is as follows

$$_{t-1}V_{x;\overline{n|}} = \frac{(l_{x+(t+1)-1} \cdot {}_{(t+1)-1}V_{x;\overline{n|}} + l_{x+(t+1)-1}P_{x;\overline{n|}})(1+i) - d_{x+(t+1)-1})}{l_{x+(t+1)}}$$

$$= \frac{(l_{x+t} \cdot {}_{t}V_{x;\overline{n|}} + l_t P_{x;\overline{n|}})(1+i) - d_{x+t})}{l_{x+t+1}} \quad (15)$$

Substitute equation (2) into equation (15), resulting in:

$$_{t+1}V_{x;\overline{n|}} = \frac{l_{x+t} \cdot ({}_{t}V_{x;\overline{n|}} + P_{x;\overline{n|}})v^{-1}}{l_{x+t+1}} - \frac{d_{x+t}}{l_{x+t+1}}$$

$$= \frac{v^{x+t+1}}{v^{x+t+1}} \cdot \frac{l_{x+t} \cdot ({}_{t}V_{x;\overline{n|}} + P_{x;\overline{n|}})v^{-1}}{l_{x+t+1}} - \frac{v^{x+t+1}}{v^{x+t+1}} \cdot \frac{d_{x+t}}{l_{x+t+1}}$$

$$= \frac{v^{x+t} l_{x+t} \cdot ({}_{t}V_{x;\overline{n|}} + P_{x;\overline{n|}})v^{-1}}{v^{x+t+1} l_{x+t+1}} - \frac{v^{x+t+1} d_{x+t}}{v^{x+t+1} l_{x+t+1}} \quad (16)$$

Substitute equations (3) and (6) into equation (16), resulting in:

$$_{t+1}V_{x;\overline{n|}} = \frac{D_{x+t} \cdot ({}_{t}V_{x;\overline{n|}} + P_{x;\overline{n|}})}{D_{x+t+1}} - \frac{C_{x+t}}{D_{x+t+1}} \quad (17)$$

Substitute $u_{x+t} = \frac{D_{x+t}}{D_{x+t+1}}$ and $k_{x+t} = \frac{C_{x+t}}{C_{x+t+1}}$ into equation (17), then it can be written as:

$$_{t+1}V_{x;\overline{n|}} = u_{x+t}\left({}_{t}V_{x;\overline{n|}} + P_{x;\overline{n|}}\right) - k_{x+t} \quad (18)$$

4. Results and Discussion

From the results of the premium reserve analysis using the Fackler method based on insured's age, the Fackler premium reserve results are obtained and presented in Table 2.

Table 2. The Fackler Premium Reserve Results

Insured's Age	Premium Amount	Premium Reserve (t=4)
26	IDR1,000,000.00	IDR204,043.47
31	IDR1,000,000.00	IDR204,749.20
32	IDR1,000,000.00	IDR204,847.64
22	IDR1,000,000.00	IDR203,504.01
28	IDR1,000,000.00	IDR204,327.41

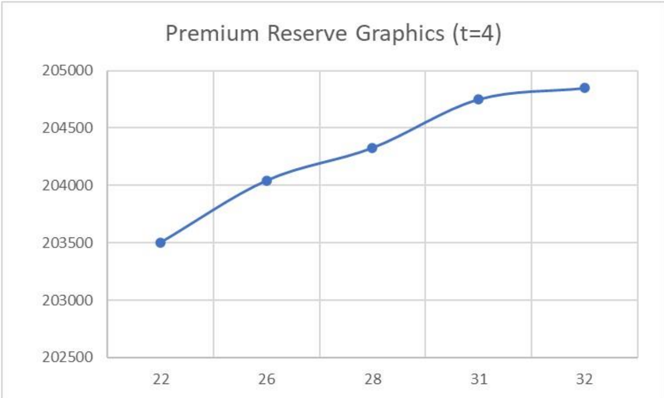

Figure 1. The Fackler Premium Reserve Results Graph

The calculations in Table (2) are derived from Equation (18), which was previously used to calculate the life annuity certain using equation (9), the single premium of endowment life insurance using equation (10), and then further calculated the annual premium of endowment life insurance using equation (12). Based on the calculation results, differences can be observed in the outcomes corresponding to different insured's age, as illustrated in figure (1).

5. Conclussion

From the calculation results of the Fackler premium reserve in figure (1), it can be concluded that as the insured's age increases when enrolling in endowment life insurance, the required premium reserve for the policy is likely to be higher. This is influenced by several factors, including the mortality risk rate present in the mortality table, causing the insurance company to need a larger premium reserve to cover this risk. Additionally, premiums at an older age are also higher due to a shorter life expectancy, resulting in higher insurance payments over a relatively shorter time period.

References

Ananda, B. D., & Sudding, F. N. F. (2023). Estimation of Premium Reserves for Last Survivor Endowment Insurance Using the New Jersey Method. *Journal of Actuarial, Finance, and Risk Management*, *2*(1), 33-44.

Futami, Takashi. (1993). Sheimei Hoken Sugaku, Jokan ("92 Revision). Japan: The Research Institute of Life Insurance Welfare. Matematika Asuransi Jiwa Bagian I. (Gatot Herliyanto). Tokyo: IncoIDRorated Foundation Oriental Life Insurance Cultural Development Center.

Gupta, R. C. (Ed.). (2011). *Insurance and risk management*. Alfa Publ..

Mishra, M. N., & Mishra, S. B. (2011). *Insurance Principles and Practice*. S. Chand Publishing.

Neumeyer, P. A., & Perri, F. (2005). Business cycles in emerging economies: the role of interest rates. *Journal of monetary Economics*, *52*(2), 345-380.

Reuß, A., Ruß, J., & Wieland, J. (2016). Participating life insurance products with alternative guarantees: reconciling policyholders' and insurers' interests. *Risks*, *4*(2), 11.

Shen, F., Liu, Y., Wang, R., & Zhou, W. (2020). A dynamic financial distress forecast model with multiple forecast results under unbalanced data environment. *Knowledge-Based Systems*, *192*, 105365.