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Risk of Ruin (ROR) Analysis in Casino Games Using Poisson Distribution

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Abstract

Gambling in casino games is an uncertain business because it creates two possibilities between the hope of winning or the risk of losing. The risks faced by casinos are usually analyzed using the Risk of Ruin (ROR). The main focus of this study is to apply the mathematical model of ROR using the Poisson distribution to model random events in gambling by considering the house advantage (a) and the law of large numbers. This study discusses the relationship between variables, such as maximum bet limits and cash flows and examines how these factors affect the risk of casino bankruptcy. In its business characteristics, casinos operate as gambling business entities and utilize the house advantage to achieve their financial benefits. House advantage indicates the profitability of the casino. However, the uncertainty of this gambling can pose a risk of bankruptcy for them. In this study, the house advantage is included in our model for several popular casino games. In addition, a set of full-range scales is defined to facilitate effective assessment of the level of risk faced by the casino, considering its regulatory context. This study also uses the binomial random walk model to describe the race between the casino and the gambler, where each step has two possible outcomes, namely winning or losing. The results of this study are expected to provide insight into the risk in calculating risk in optimizing betting decisions and reducing the risk of bankruptcy.

Keywords: Risk of Ruin (ROR), house advantage, poisson distribution, binomial random walk.

1. Introduction

Gambling is one of the parts that is inherent in the history of human civilization. In addition to being done as entertainment, gambling also affects psychological, social, and even economic aspects. Gambling is usually done in places where various forms of gambling games and gambling-related entertainment are available, such as casinos. Like businesses in general, casinos will always calculate profitability and risk and try to balance both. Profitability is determined by the house advantage and maximum betting limits. The maximum betting limit is a policy and power that can be set by the casino to protect its gambling business. The higher the house advantage and maximum betting limit, the higher the casino's profit. However, the higher the probability of the casino experiencing the threat of bankruptcy. In fact, there is a group of gamblers known as "high rollers" who can place bets worth tens of thousands of US dollars. Even "whales", namely high rollers with the highest spending who can bet up to hundreds of thousands of dollars per game, involve betting restrictions in a certain amount against the casino. Therefore, they may be able to bankrupt small casinos. To minimize the risk, casinos need to sacrifice high profits and manage their bankroll by limiting the maximum bet amount. A number of popular gambling games can be found in casinos, such as baccarat, roulette, blackjack, Sic Bo, and craps. These games are basically categorized into games of skill and games of luck. Games of skill require gamblers to make decisions during the game that will affect the course of the game and thus affect their payout rate (also called house advantage) to some extent. In contrast, games of luck are games that rely on luck where gamblers do not need to make decisions other than determining the amount of bets, and the payout rate will remain the same. Examples include baccarat, roulette, Sic Bo, and craps. House advantage (a) reflects the percentage chance of winning and the profit that the casino gets for a particular bet on a game in the long run. Each game has a different way of betting. For example, in the game of baccarat there are at least four bets, including banker, player, tie, etc. Each bet has a probabilistic model that determines the probability that it will occur in a game[4]. To calculate the house advantage of a game, we must determine the probability of a particular event occurring and the amount the casino will pay out when the bet wins (or the payout rate). In terms of risk assessment, the house advantage certainly helps reduce the risk of bankruptcy, but we have no clue to what extent the risk can be reduced. The number of trials also determines the risk of a casino business. According to the law of large numbers (aka the 1st Law of Probability), if a large number of trials are conducted for a random event, the average of the trial outcomes should be close to the expected value or theoretical outcome. This is an important theorem underlying probability. With such a large sample size, it is likely that the average of the trials will be close to the expected value, thus giving the casino its desired profit. Theoretically, the more trials are conducted, the lower the risk. The number of trials ultimately determines the outcome, which is closely related to the casino's bankroll and the maximum bet limit allowed. So, the number of trials (n) is the division of the bankroll by the maximum bet limit.

2. Theoretical basis

The bets made between gamblers and casinos depend on probability and the Risk of Ruin (ROR) always decreases in the long run when the casino has a good house advantage and positive overall expectation. For the calculation of ROR, the higher the bet, the higher the probability of the casino going bankrupt for a certain bankroll (initial capital). Then the risk will decrease as the amount of bets increases and the casino's bankroll is larger than the gambler's so that there is a possibility that the gambler can catch up with the casino's bankroll which decreases exponentially in the number of steps n, if the casino has a house advantage (a) on each bet. This can provide a general way to estimate the guarantee of sustainability with a certain number of games, but does not take into account the probability of winning (p) of the casino in the game.

3. Objects and Methods

3.1 Object

Conventionally, each game in a casino is played separately from the other games. Each type of gambling has rules that detail all aspects of the game, including the handling of the game to achieve a predetermined house advantage, the type of equipment used, and the procedures to be followed, etc. The purpose of these rules is to ensure that the house advantage is achieved in the long run. As a result, the probability of winning and the house advantage can be calculated based on these rules. By applying this model, we only need to provide the p and a values of various games, then from the model we can calculate the probability results. Some types of casino games depend entirely on luck, no skill or strategy can change the odds. These types of games include roulette, craps, baccarat, keno, and big six. The best odds for the player occur when the house advantage percentage is smaller. The house advantage percentage reflects the average percentage of total bets won by the casino in the long run. The smaller the house advantage, the less profit the casino expects from each player's bet. Of the several types of casino games, baccarat and craps provide the best odds, where baccarat produces a house advantage of 1.2% while craps produces a house advantage of less than 1%. Big six puts a heavier burden on the player with a house advantage of more than 7%, even reaching almost 20% for bets on the "yellow" option. Keno is considered a casino scam with a house advantage approaching 30%.

In addition, there is also a type of casino game, namely blackjack. Blackjack is one of the most preferred and popular table games worldwide because it combines elements of luck and strategic decisions. Although the blackjack house advantage varies slightly depending on the rules and number of decks, players who follow basic strategy experience little to no disadvantage in single-deck games. In a typical six-deck game, the house advantage is only about 0.5%. Player-favorable rule variations include the use of fewer decks, the dealer standing on soft seven (adds 0.2% to the player's advantage), doubling after splitting (0.14%), late surrender (0.06%), and early surrender (rare, but worth 0.24%). However, on average, the player ends up giving the casino an edge of 2.0% due to mistakes and deviations from basic strategy.

The following table shows the Risk of Ruin (ROR) probabilities obtained by applying the proposed model to each bet based on the type of casino game.

Table 1: ROR probabilities obtained by applying the proposed model to each Baccarat bet.

Bet	p	a	n									
			1	2	3	4	5	6	7	8	9	10
Banker	45.9%	1.06%	6.53×10^{-1}	4.80×10^{-1}	3.61×10^{-1}	2.74×10^{-1}	2.10×10^{-1}	1.61×10^{-1}	1.24×10^{-1}	9.59×10^{-2}	7.42×10^{-2}	5.75×10^{-2}
Player	44.6%	1.24%	6.40×10^{-1}	4.61×10^{-1}	3.41×10^{-1}	2.54×10^{-1}	1.91×10^{-1}	1.45×10^{-1}	1.09×10^{-1}	8.31×10^{-2}	6.32×10^{-2}	4.81×10^{-2}
Series	9.5%	14.36%	1.27×10^{-1}	2.00×10^{-2}	3.26×10^{-3}	5.43×10^{-4}	9.18×10^{-5}	1.57×10^{-5}	2.74×10^{-6}	4.85×10^{-7}	8.74×10^{-8}	1.61×10^{-8}

Table 2: ROR probabilities obtained by applying the proposed model to each Sic Bo bet.

Bet	p	a	n = 1	2	3	4	5	6	7	8	9	10
Small large	48.61%	2.78%	6.72×10^{-1}	5.05×10^{-1}	3.88×10^{-1}	3.01×10^{-1}	2.35×10^{-1}	1.84×10^{-1}	1.45×10^{-1}	1.14×10^{-1}	9.02×10^{-2}	7.13×10^{-2}
Dice Combination	13.90%	2.8%	2.38×10^{-1}	6.84×10^{-2}	2.04×10^{-2}	6.17×10^{-3}	1.88×10^{-3}	5.77×10^{-4}	1.77×10^{-4}	5.47×10^{-5}	1.69×10^{-5}	5.22×10^{-6}
Double Specific	7.41%	33.3%	6.05×10^{-2}	4.67×10^{-3}	4.00×10^{-4}	3.89×10^{-5}	4.47×10^{-6}	6.06×10^{-7}	9.32×10^{-8}	1.55×10^{-8}	2.69×10^{-9}	4.76×10^{-10}
Double Specific	2.78%	30.6%	1.84×10^{-2}	4.35×10^{-4}	1.15×10^{-5}	3.58×10^{-7}	1.44×10^{-8}	7.49×10^{-10}	4.61×10^{-11}	3.08×10^{-12}	2.13×10^{-13}	1.49×10^{-14}
Certain Triple	0.46%	30.1%	1.82×10^{-3}	4.24×10^{-6}	1.12×10^{-8}	4.05×10^{-11}	2.57×10^{-13}	2.44×10^{-15}	$< 1.0 \times 10^{-15}$	$< 1.0 \times 10^{-15}$	$< 1.0 \times 10^{-15}$	$< 1.0 \times 10^{-15}$

Table 3: ROR probabilities obtained by applying the proposed model to each Big Six bet.

Bet	p	a	n = 1	2	3	4	5	6	7	8	9	10
Orange	46.15%	7.69%	6.27×10^{-1}	4.42×10^{-1}	3.20×10^{-1}	2.34×10^{-1}	1.72×10^{-1}	1.27×10^{-1}	9.40×10^{-2}	6.98×10^{-2}	5.19×10^{-2}	3.87×10^{-2}
Purple	25.08%	7.69%	3.78×10^{-1}	1.68×10^{-1}	7.76×10^{-2}	3.63×10^{-2}	1.71×10^{-2}	8.08×10^{-3}	3.84×10^{-3}	1.83×10^{-3}	8.75×10^{-4}	4.19×10^{-4}
Green	15.38%	7.69%	2.39×10^{-1}	6.91×10^{-2}	2.07×10^{-2}	6.28×10^{-3}	1.92×10^{-3}	5.93×10^{-4}	1.84×10^{-4}	5.70×10^{-5}	1.78×10^{-5}	5.54×10^{-6}
Blue	7.69%	15.38%	9.88×10^{-2}	1.21×10^{-2}	1.54×10^{-3}	2.00×10^{-4}	2.65×10^{-5}	3.58×10^{-6}	4.49×10^{-7}	6.97×10^{-8}	1.01×10^{-8}	1.52×10^{-9}
Yellow	3.85%	19.23%	4.03×10^{-2}	2.03×10^{-3}	1.08×10^{-4}	5.91×10^{-6}	3.38×10^{-7}	2.04×10^{-8}	1.33×10^{-9}	9.39×10^{-11}	7.25×10^{-12}	6.04×10^{-13}
Logo 1 / Logo 2	1.92%	11.54%	2.41×10^{-2}	7.21×10^{-4}	2.25×10^{-5}	7.18×10^{-7}	2.31×10^{-8}	7.56×10^{-10}	2.50×10^{-11}	8.42×10^{-13}	2.90×10^{-14}	$< 1.0 \times 10^{-15}$

Table 4: ROR probabilities obtained by applying the proposed model to each Roulette bet.

Bet	p	a	n = 1	2	3	4	5	6	7	8	9	10
Red / Black	$18/37$	2.7%	6.73×10^{-1}	5.06×10^{-1}	3.89×10^{-1}	3.02×10^{-1}	2.36×10^{-1}	1.85×10^{-1}	1.46×10^{-1}	1.15×10^{-1}	9.09×10^{-2}	7.20×10^{-2}
Corner	$4/37$	2.7%	1.88×10^{-1}	4.32×10^{-2}	1.03×10^{-2}	2.48×10^{-3}	6.04×10^{-4}	1.48×10^{-4}	3.63×10^{-5}	8.93×10^{-6}	2.20×10^{-6}	5.43×10^{-7}
Split	$2/37$	2.7%	9.61×10^{-2}	1.14×10^{-2}	1.40×10^{-3}	1.76×10^{-4}	2.21×10^{-5}	2.81×10^{-6}	3.57×10^{-7}	4.55×10^{-8}	5.80×10^{-9}	7.42×10^{-10}
Single	$1/37$	2.7%	4.81×10^{-2}	2.87×10^{-3}	1.78×10^{-4}	1.12×10^{-5}	7.15×10^{-7}	4.57×10^{-8}	2.93×10^{-9}	1.88×10^{-10}	1.21×10^{-11}	8.42×10^{-13}

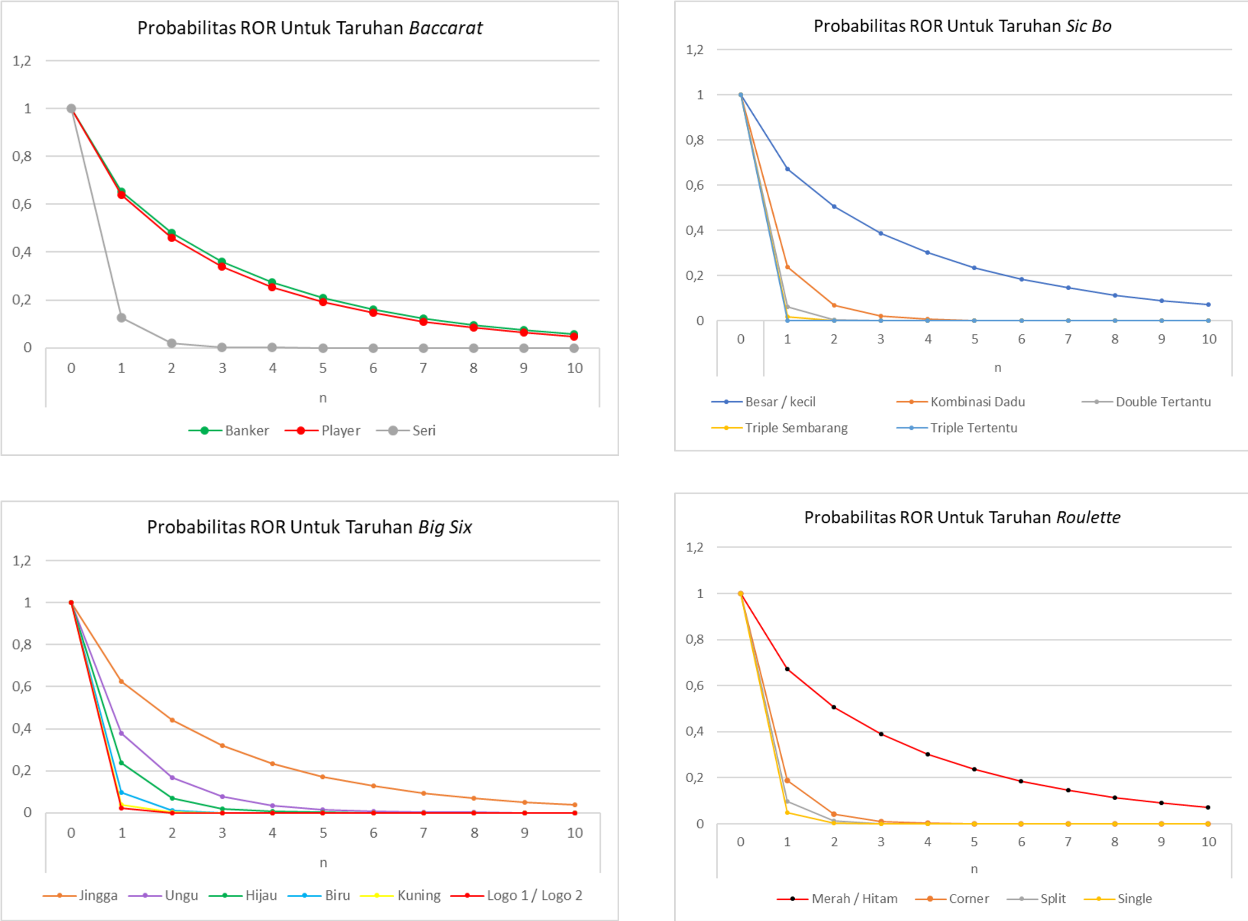

Figure 1: ROR probability of the proposed model on each casino game bet

3.2 Metode

3.2.1 Binomial Distribution

The binomial distribution is a probability distribution associated with repeated trials that are discrete in nature, that is, have two possible outcomes, usually referred to as "success" and "failure." Each trial is considered independent of each other, and the binomial distribution gives the probability of a given number of successes occurring in a fixed number of trials.

The assumptions used in the binomial experiment are:

a) Each experiment has two possible outcomes, namely Success and Failure, which are independent of each other.
b) The probability of success is indicated by the symbol p which remains constant from one experiment to the next and the probability of failure is indicated by the symbol q.
c) The experiments as many as n times are independent, meaning that the results of each experiment do not affect the results of other experiments.

The magnitude of the probability value of each x successful event from n experiments is indicated by the probability of success p and the probability of failure q.

Definition of Binomial probability distribution:

$$b(x,n,p) = \binom{n}{x} p^x q^{n-x} = \frac{n!}{x!\,(n-x)!} p^x q^{n-x} \quad (1)$$

with:

$p = 1 - q =$ probability of success
$q =$ probability of failure

n = total number of trials
x = number of successes out of n trials

The binomial distribution is commonly used in the context of repeated experiments that have two possible outcomes, such as the flip of a coin (heads or tails), the success or failure of an exam, or a bet in a casino with a win or loss outcome.

Some properties of the Binomial distribution are as follows:

Mean $\mu = n\,p$
Varian $\sigma^2 = n\,p\,q$
Standard Deviation $\sigma = \sqrt{n\,p\,q}$

3.2.2 Poisson distribution

The Poisson distribution is the number of events that occur in a certain time interval or in a certain area. If a binomial experiment is carried out approaching infinity $(n \to \infty)$, and the probability of success is very small $(p \to 0)$, then the binomial distribution will approach the Poisson distribution with parameters $np = \lambda$.

If $X \sim b(x, n, p)$, so

$$p(x) = \binom{n}{x} p^x (1-p)^{n-x} \tag{2}$$

If $n \to \infty$, so

$$\begin{aligned}\lim_{n\to\infty} p(x) &= \lim_{n\to\infty} \binom{n}{x} p^x (1-p)^{n-x} \\ &= \lim_{n\to\infty} \frac{n!}{x!\,(n-x)!} p^x (1-p)^{n-x}\end{aligned}$$

Because $np = \lambda$, so $p = \frac{\lambda}{n}$, so that

$$\begin{aligned}\lim_{n\to\infty} p(x) &= \lim_{n\to\infty} \frac{n(n-1)(n-2)\dots(n-x+1)\cancel{(n-x)}!}{x!\,\cancel{(n-x)}!} \left(\frac{\lambda}{n}\right)^x \left(1-\frac{\lambda}{n}\right)^{n-x} \\ &= \lim_{n\to\infty} \frac{n(n-1)(n-2)\dots(n-x+1)}{n^x} \frac{\lambda^x}{x!} \frac{\left(1-\frac{\lambda}{n}\right)^n}{\left(1-\frac{\lambda}{n}\right)^x} \\ &= \lim_{n\to\infty} \frac{n}{n} \frac{(n-1)}{n} \frac{(n-2)}{n} \frac{(n-3)}{n} \dots \frac{(n-x+1)}{n} \frac{\lambda^x}{x!} \frac{\left(1-\frac{\lambda}{n}\right)^n}{\left(1-\frac{\lambda}{n}\right)^x} \\ &= \lim_{n\to\infty} 1\left(1-\frac{1}{n}\right)\left(1-\frac{3}{n}\right)\left(1-\frac{3}{n}\right)\dots\left(1-\frac{(x-1)}{n}\right) \frac{\lambda^x}{x!} \frac{\left(1-\frac{\lambda}{n}\right)^n}{\left(1-\frac{\lambda}{n}\right)^x}\end{aligned}$$

Remember thatlim$_{n\to\infty} \frac{a}{n} = 0$, so that

$$\begin{aligned}\lim_{n\to\infty} p(x) &= \frac{\lambda^x}{x!} \lim_{n\to\infty} 1(1-0)(1-0)(1-0)\dots(1-0) \frac{\left(1-\frac{\lambda}{n}\right)^n}{(1-0)^x} \\ &= \frac{\lambda^x}{x!} \lim_{n\to\infty} \left(1+\frac{(-\lambda)}{n}\right)^n\end{aligned}$$

Remember that

$$e = \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^x$$

For example $x = \frac{-n}{\lambda}$, so

$$\lim_{n\to\infty} \left(1+\frac{1}{x}\right)^{-\lambda x} = \left[\lim_{n\to\infty} \left(1+\frac{1}{x}\right)^{-x}\right]^{\lambda} = e^{-\lambda}$$

So we get the formula for the Poisson distribution.

$$\lim_{n\to\infty} p(x) = \frac{\lambda^x e^{-\lambda}}{x!} \tag{3}$$

So it can be concluded that when n is very large, the Binomial distribution will become a Poisson distribution.

3.2.3 *Expected Value*

To calculate the house advantage of a bet in a casino game, we must determine the probability of a particular event occurring and the amount the casino will pay out for a winning bet or the payout rate. This is related to the expected value (μ). Expected value (μ) can be used to calculate how much money a player can expect to win or lose in the long run on a particular bet. This can be shown in the following formula:

$$\mu = \sum (P(X_i) \times X_i) \tag{4}$$

with:

X_i = gambling incident

$P(X_i)$ = probability X_i

Simply put, μ is the average value of a random variable over the long run. In the gambling business, a negative μ value is usually expected because the advantage is on the casino's side or the house advantage. The house advantage can be calculated by the following formula:

$$a = -\mu \times 100\% \tag{5}$$

Where μ it is used to handle the evaluation of the feasibility of probabilistic choices in this case.

3.2.4 *Risk of Ruin* (ROR)

In determining the Risk of Ruin (ROR) at a given time, it is important to determine how many times the casino can risk a loss before going bankrupt (n). This can be done using a stochastic process approach involving n, which is expressed as follows:

$$n \leftarrow \begin{cases} n+1, & casino\ winnings \\ n-1, & gambler's\ win \end{cases} \tag{6}$$

The casino will go bankrupt if $n \leq 0$, we assume the probability of the casino going bankrupt with input n is $Pro(n)$. p indicates the probability of a gambler winning once in a game, while $(1-p)$ indicates the probability of the casino winning once in the same game. This case can be seen as a positive probability in a time interval and a changing frequency. So it can be concluded that the Risk of Ruin (ROR) is in accordance with the Poisson distribution.

Assume a gambler with unlimited credit has the opportunity to play an infinite number of games trying to bankrupt the casino in a few bets. Meanwhile, the probability of the gambler winning is always p. We define n as follows:

$$n = \left\lfloor \frac{Bankroll}{MaxBet} \right\rfloor \tag{7}$$

where MaxBet is the upper limit of the gambler's bet that has been set in a game, so that the distance between the gambler and his goal is n. The race between the casino and the gambler can be considered as a Binomial Random Walk problem about estimating $Pro(n)$. n is the distance from the starting point, which means that the casino has won n times since the beginning so that it can withstand the gambler winning n times before going bankrupt. Based on equation (6), the event of a casino winning causes n to move forward by 1, or widen the distance by 1, denoted as $(n \leftarrow n+1)$, and the event of a gambler winning causes n to be chased by 1, or narrow the distance by 1, denoted as $(n \leftarrow n-1)$. Then, we can calculate the probability that the casino goes bankrupt, which is the probability that n becomes 0:

- p, states the probability of a gambler winning in a single round
- p^{n-k} states the probability that a gambler always wins continuously from k to the goal n.

In this situation, the probability of the casino going bankrupt can be stated as follows:

$$Pro(n) = \begin{cases} p^{n-k}, & k \leq n \\ 1, & k > n \end{cases} \tag{8}$$

Reconsidering the probability of casino bankruptcy decreases exponentially as the distance n increases. Since p<1 (the probability of a gambler winning in a single round) will decrease as the distance increases. In addition, it is necessary to determine n that the casino needs to ensure, so that a gambler with infinite credit will not win the casino with the money given.

3.2.5 Poisson Distribution for Risk of Ruin (ROR)

We assume that the gambler is allowed to play t times (as many as possible) in a certain period of time, and the probability of the gambler winning k times in the same period of time, so using the Poisson distribution, it can be expressed as follows:

$$\lim_{n\to\infty} \binom{t}{k} p^k (1-p)^{t-k} \tag{9}$$

Since this is a binomial analysis as mentioned earlier, with t being the number of trials and p being the probability of an equal game, the expected value (μ) is $\mu = t \times q$, so equation (9) becomes:

$$\lim_{n\to\infty} \binom{t}{k}\left(\frac{\mu}{t}\right)^{k}\left(1-\frac{\mu}{t}\right)^{t-k} = \frac{\mu^{k} e^{-\mu}}{k!} \tag{10}$$

Next, the probability of the variable k can be used in the Poisson distribution. In this case, to obtain the probab of the Risk of Ruin (ROR) for each n that can still be pursued by the gambler, we can multiply equation (10) by each series of the number of rounds that the casino can hold with the provision that each distance k, indicated by equation (8).

$$Pro(n) = \sum_{k=0}^{\infty}\left(\frac{\mu^{k} e^{-\mu}}{k!}\right)\begin{cases} p^{n-k}, & k \le n \\ 1, & k > n \end{cases}$$

$$= e^{-\mu}\left(\sum_{k=0}^{n}\frac{\mu^{k}}{k!}p^{n-k} + \sum_{k=0}^{\infty}\frac{\mu^{k}}{k!}\right) \tag{11a}$$

$$= e^{-\mu}\left(\sum_{k=0}^{n}\frac{\mu^{k}}{k!}p^{n-k} + \sum_{k=0}^{\infty}\frac{\mu^{k}}{k!} - \sum_{k=0}^{n}\frac{\mu^{k}}{k!}\right) \tag{11b}$$

$$= e^{-\mu}\left(\sum_{k=0}^{n}\frac{\mu^{k}}{k!}p^{n-k} + e^{\mu} - \sum_{k=0}^{n}\frac{\mu^{k}}{k!}\right) \tag{11c}$$

(11d)

Here, equation (11c) is rearranged to avoid infinite iteration summation in equation (11b), and substituted by the Taylor expansion of the exponential function $e^{-\mu} = \sum_{k=0}^{\infty}\frac{\mu^{k}}{k!}$ between equation (11c) and equation (11d). So we get:

$$Pro(n) = 1 - e^{-\mu}\sum_{k=0}^{n}\frac{\mu^{k}}{k!}(1-p)^{n-k} \tag{12}$$

With equation (12), we can estimate the number of rounds (n) better where the gambling risk is reduced to a negligible level. Another advantage of equation (12) is that the probability of failure is zero and in this case, if the gambler continues to bet in the hope of catching up from a long distance $(n-k)$ behind the casino's bankroll, then he has a positive probability of total ruin.

3.2.6 House Advantage

Like any other business, casinos are always concerned with profitability and risk and try to achieve the right balance between the two. On the other hand, the probability model in equation (12) is not integrable because the number of gamblers is infinite in real life. Therefore, casinos must manage unfavorable situations with strategies that can be profitable. To reduce risk, casinos need to sacrifice high profits by considering their bankroll and maximum bet limits. In general, this can be profitable for the casino because the theoretical win is expressed as the hold multiplied by the house advantage, denoted by $(a > 0)$. This value is usually in a varying percentage range, ranging from quite small ($< 1\%$ for blackjack) to quite large ($> 25\%$ for keno). These numbers represent the average amount of all bets that a player will lose in the long run. Therefore, the race between the casino's win and the gambler's win in equation (6) should be formulated as:

$$n \leftarrow \begin{cases} n + (1+a), & \textit{casino winnings} \\ n - 1, & \textit{kemenangan penjudi} \end{cases} \tag{13}$$

Here, a is the house advantage related to the casino's profitability and the modified Poisson distribution, expressed as follows:

$$Pro(n) = \sum_{k=0}^{\infty}\left(\frac{\mu^{k} e^{-\mu}}{k!}\right)\begin{cases} p^{n(1-a)-k}, & k \le n \\ 1, & k > n \end{cases} \tag{14}$$

Any positive value of the house advantage (a) used will not affect the probability of a gambler winning in a given time period. Furthermore, the house advantage increases the forward speed n, so the distance between the casino and the gamble is n(1+a)-k and the final probability model we derive is as follows:

$$Pro(n) = 1 - e^{-\mu} \sum_{k=0}^{\infty} \frac{\mu^k}{k!} \left(1 - p^{n(1-a)-k}\right) \quad (15)$$

4. Result and Discussion

We used several other ROR models, such as *Coolidge* [8], *Kaufmann* [33] and *Ralph-Vince* [34] and negative binomial regression (NBRM) models [35] as a comparison with the model we chose to analyze. It also shows a model that ignores *the house advantage* () and is assumed to be a risk that the casino can still bear before going bankrupt. In the $a = 0n = 10$*Coolidge* model, independent control variables of the same value are used in the *Kaufmann* and *Ralph-Vince* models, and their dependent variables (and win rates) have been described with a more plausible ordinal model. Similar to the model we chose, we expanded the model and took into account the *phouse advantage* rather than the current winning percentage. The probability of winning and *the bankroll* or budget are considered independent variables on each model.

In empirical data simulations, *the Coolidge* model gives the worst predictions by ignoring p variables and variables and is not used as different input variablesap because and is considered to be derived directly from each other so that the prediction results are very simple and unclear. In the *apKaufmann* and *Ralph-Vince* methods, the results are more accurate because they both analyze the probability of winning and consider the percentage of risk. However, it is still not perfect because the risk percentage requires the amount of risk/return and the win rate, which will change over time and is not. suitable for ROR estimation. But in reality, rather than predicting the current casino's win and loss ratio, it is better to consider ROR predictions in the next round. Taking these factors into account can make the formula very complicated, and this is where Poisson's distribution model comes into play n.

It should be noted here that when comparing the performance of different models, we should not include casino advantages () into the NBRM model. This is mainly because the model is a computational model and is typically designed to model interval-scale data, not ordinal-scale data. In addition, unlike the features and, the percentage of risk varies from person to person and does not affect the ROR. In contrast, the risk percentage usually affects and indirectly affects MaxBet. Therefore, it does not make sense to include this information directly into the NBRM and the model we propose $n \times aap$.

Table 5: Comparison with other models.

Game	Taruhan	*Coolidge*	*Kaufmann*	*Ralph-Vince*	NBRM	(12)	(13)
Baccarat	Banker	4.74×10^{-1}	7.87×10^{-3}	6.13×10^{-2}	6.42×10^{-2}	6.18×10^{-2}	5.75×10^{-2}
	Player	4.69×10^{-1}	6.78×10^{-3}	5.54×10^{-2}	6.13×10^{-2}	5.25×10^{-2}	4.81×10^{-2}
	Seri	2.07×10^{-1}	6.18×10^{-3}	4.76×10^{-4}	2.14×10^{-7}	3.03×10^{-7}	1.61×10^{-8}
Sic Bo	Big/Small	4.32×10^{-1}	1.06×10^{-2}	7.52×10^{-2}	6.81×10^{-2}	8.46×10^{-2}	7.13×10^{-2}
	Dice Combinations	4.31×10^{-1}	3.11×10^{-5}	1.43×10^{-3}	2.59×10^{-6}	8.88×10^{-6}	5.22×10^{-6}
	Specific Double	5.34×10^{-2}	2.19×10^{-6}	2.35×10^{-4}	1.87×10^{-8}	3.10×10^{-8}	4.76×10^{-10}
	Triple Arbitrary	6.48×10^{-2}	3.95×10^{-8}	1.54×10^{-5}	8.45×10^{-11}	2.70×10^{-12}	1.49×10^{-14}
	Specific Triple	6.71×10^{-2}	2.82×10^{-11}	1.12×10^{-7}	5.09×10^{-12}	$< 1.0 \times 10^{-15}$	$< 1.0 \times 10^{-15}$
Big Six	Orange	3.23×10^{-1}	8.10×10^{-3}	6.25×10^{-2}	6.13×10^{-2}	6.37×10^{-2}	3.87×10^{-2}
	Purple	3.23×10^{-1}	4.23×10^{-4}	8.41×10^{-3}	1.82×10^{-3}	1.09×10^{-3}	4.19×10^{-4}
	Green	3.23×10^{-1}	4.82×10^{-5}	1.92×10^{-3}	2.91×10^{-5}	2.11×10^{-5}	5.54×10^{-6}
	Blue	1.93×10^{-1}	2.56×10^{-6}	2.61×10^{-4}	1.82×10^{-9}	4.37×10^{-8}	1.52×10^{-9}
	Yellow	1.47×10^{-1}	1.48×10^{-7}	3.78×10^{-5}	1.07×10^{-12}	6.30×10^{-11}	6.04×10^{-13}
	Logo 1/2	2.51×10^{-1}	8.83×10^{-9}	5.55×10^{-6}	4.94×10^{-13}	7.26×10^{-14}	$< 1.0 \times 10^{-15}$
Roulette	Red/Black	4.34×10^{-1}	1.07×10^{-2}	7.54×10^{-2}	6.63×10^{-2}	8.50×10^{-2}	7.20×10^{-2}
	Corner	4.34×10^{-1}	1.07×10^{-5}	6.90×10^{-4}	2.22×10^{-8}	9.72×10^{-7}	5.43×10^{-7}
	Split	4.34×10^{-1}	5.95×10^{-7}	9.71×10^{-5}	1.55×10^{-10}	1.61×10^{-9}	7.42×10^{-10}
	Single	4.34×10^{-1}	3.52×10^{-8}	1.42×10^{-5}	8.65×10^{-13}	2.05×10^{-12}	7.80×10^{-13}

Table 5 above is the result of a *benchmark* comparison between several ROR models and the model we chose with or without house *advantage* considerations. All models use the same provisions, i.e. the derivatives of the control variables (and) are essentially the same in all these models, where the greater the value, the greater the ROR generated because. However, the greater the value$app \frac{\partial}{\partial a} Pro(n) > 0a$, the smaller the ROR because. $\frac{\partial}{\partial a} Pro(n) < 0$

Compared to *Kaufmann* and *Ralph-Vince*, the results on *the Coolidge* model tend to be conservative, more comprehensive, and easier to identify, but the higher ROR values despite the gamblers' low win rate plus the *Coolidge* model is not perfect because the current number of wins/losses must be calculated before prediction and cannot be made due to different results every time. For example, when a tie occurs on a *baccarat* bet with, the ROR value is obtained 2. Another limitation is that an upper limit must be defined, which is not possible. For the NBRM model, the results are almost similar to the selected model, namely Equation (12), with the difference in the house $p = 9.5\%.07 \times 10^{-1}n$*advantage* consideration. This is because the model also uses the binomial distribution of a finite sample and the chosen model describes the distribution of an infinite sample, thus giving the probability of obtaining the entire event in the population. In addition, the prediction of destruction can be further minimized when considering the impact of profits (). The results of the simulations using these models are considered quite understandable and these models can roughly determine how each variable affects the final result $a > 0$.

After knowing the results of the ROR prediction, we must be able to know if the results obtained can be made to determine the level of risk in a game. To measure it, then we can use the logarithmic scale function to be able to analyze it, by

$$ROR(x) = \log x - \log s = \log \frac{x}{s} \tag{16}$$

where x is the ROR obtained and s be 10^{-10} as a number to limit.

The use of logarithmic scales states the value of the index with high accuracy where each step makes the probability increase tenfold with the maximum value of the scale being 10. The ROR scale states that casinos will not go bankrupt≤ 1.25, $1.25 < ROR(x) \leq 2.50$ describing casinos going bankrupt by 1 in 600 million gamblers, $2.50 < ROR(x) \leq 3.75$ describing a casino going bankrupt by 1 in 30 million gamblers, $3.75 < ROR(x) \leq 5.00$ describing a casino going bankrupt by 1 in 1,750,000 gamblers, $5.00 < ROR(x) \leq 6.25$ Describe a casino going bankrupt by 1 in 100 thousand gamblers, $6.25 < ROR(x) \leq 7.50$ depicts a casino going bankrupt by 1 in 5000 gamblers, and $7.50 < ROR(x) \leq 10.0$ depicts a casino going bankrupt by every gambler.

By applying equation (16) with Table 1, Table 2, Table 3, and Table 4, the ROR scale is obtained as Table 6, Table 7, Table 8, and Table 9:

Table 6: Scale ROR on Baccarat bets.

Bet	p	a	n									
			1	2	3	4	5	6	7	8	9	10
Banker	45.9%	1.06%	9.815	9.681	9.558	9.438	9.322	9.207	9.093	8.982	8.870	8.760
Player	44.6%	1.24%	9.806	9.664	9.533	9.405	9.281	9.161	9.037	8.920	8.801	8.682
Seri	9.5%	14.36%	9.104	8.301	7.513	6.735	5.963	5.196	4.438	3.686	2.942	2.207

Table 7: ROR scale on Sic Bo bets

Bet	p	a	n									
			1	2	3	4	5	6	7	8	9	10
Big/small	48.61%	2.78%	9.827	9.703	9.589	9.479	9.371	9.265	9.161	9.057	8.955	8.853
Dice Combinations	13.90%	2.8%	9.377	8.835	8.310	7.790	7.274	6.761	6.248	5.738	5.228	4.718
Specific Double	7.41%	33.3%	8.782	7.669	6.602	5.590	4.650	3.782	2.969	2.190	1.430	0.678
Triple Arbitrary	2.78%	30.6%	8.265	6.638	5.061	3.554	2.158	0.874	<0.00	<0.00	<0.00	<0.00
Specific Triple	0.46%	30.1%	7.260	4.627	2.049	<0.00	<0.00	<0.00	<0.00	<0.00	<0.00	<0.00

Tabel 8: ROR scale on *Big Six*.

Bet	p	a	n									
			1	2	3	4	5	6	7	8	9	10
Orange	46.15%	7.69%	9.797	9.645	9.505	9.369	9.236	9.104	8.973	8.844	8.715	8.588
Purple	25.08%	7.69%	9.577	9.225	8.890	8.560	8.233	7.907	7.584	7.262	6.942	6.622
Green	15.38%	7.69%	9.378	8.839	8.316	7.798	7.283	6.773	6.265	5.756	5.250	4.744
Blue	7.69%	15.38%	8.995	8.083	7.188	6.301	5.423	4.554	3.652	2.843	2.004	1.182
Yellow	3.85%	19.23%	8.605	7.307	6.033	4.772	3.529	2.310	1.124	<0.00	<0.00	<0.00
Logo 1/2	1.92%	11.54%	8.382	6.858	5.352	3.856	2.364	0.879	<0.00	<0.00	<0.00	<0.00

Tabel 9: Scale ROR on *Roulette* bets.

Bet	p	a	n									
			1	2	3	4	5	6	7	8	9	10
Red / Black	$18/37$	2.7%	9.828	9.704	9.590	9.480	9.373	9.267	9.164	9.061	8.959	8.857
Corner	$4/37$	2.7%	9.274	8.635	8.013	7.394	6.781	6.170	5.560	4.951	4.342	3.735
Split	$2/37$	2.7%	8.983	8.057	7.146	6.246	5.344	4.449	3.553	2.658	1.763	0.870
Single	$1/37$	2.7%	8.682	7.458	6.250	5.049	3.854	2.660	1.467	0.274	<0.00	<0.00

4. Conclusion

The use of Poisson distribution to predict ROR helps to add higher accuracy coupled with predetermined variables, namely the probability of winning (p), house advantage (a), and maximum bet prediction (n). This study adds to the knowledge by exploring the relationship between money flow and maximum bet limits that affect the risk of casino bankruptcy. Through the theory of risk awareness and social support, a typology scale measure of ROR is developed and can provide a clear picture of the impact of decisions on the risk of bankruptcy, indicating that casinos need to understand the house advantage. In terms of gambling, this study provides valuable information to identify high-risk situations in a timely manner. From the perspective of responsible gambling advocacy, these findings provide a visual and concrete guide for addicted gamblers who abandon their rationality and are trapped in a hope that they can make quick profits and believe in myths about formulas to beat the casino. In conclusion, the casino has a much larger bankroll advantage compared to individual gamblers, and makes further losses for gamblers, so the longer the series of bets, the greater the chance of the casino winning without any advantage on the gambler's side.

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