



# The Estimation of relationship between actuarial rate of return, Maturity and Coupon: The case of Tunisia

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## Abstract

The aim of this paper is to study the Estimation of the relationship between the actuarial rate of return, Maturity and Coupon issued by the Tunisian banking and financial institutions knowing that it is a significant component of financing. So, this relationship issue is an essential link of investments. The studying of yield bond is an important first step. For many types of financial research. To date, this research has focused on the bond yield for institutions, with the exception of works by institution, financial and banking. There regression models are tested by different methods. The first model is based only on relationship between Actuarial Rate of Return, maturity and coupon. The other models are prominent in the published literature on the bond yield.

*Keywords:* Coupon, maturity, actuarial rate of return.

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## 1. Introduction

The liberalization of Tunisia's interest rates increases the independence of financial and banking institutions in setting interest rates. The volatile interest rates are a source of banking and financial risks that must be managed to take advantage of market opportunities and avoid capital losses. The interest rate risk represents market risk, which has a very significant impact on the returns of various financial assets. Indeed, bond borrowing is one way of ensuring the liquidity of financial and banking institutions. Consequently, several countries have recourse to bond loans, including Tunisia, whose economy is fragile. The objectives of bond issuance are: to diversify bank products more and more, since bond borrowing is also a component of net bank income, it is a securities transaction. The nature of bond activity is part of banking policy aimed at countering the increase in its commitments and reaching targeted market shares by moving towards the mobilization of stable resources in the medium and long term.

## 2. Literature Riview

The term structure of the interest rates on bonds that differ in the timing of interest payments (voucher) and maturities. Many tests of the theory of the structure of the term were developed through the relationship between the actuarial rate of return (ARR) and the voucher and the maturity of the yield bonds. Many researchers focused in the relationship between interest rate and bond yield. We mention the interpretive theories of the term interest rate structure. The purpose of the study of the future structure of interest rates is to explain the differences between short-term interest rates and long-term rates, i.e. the interpretation of changes in interest rates by term has a certain proactive assurance. To analyze this structure, an investor needs interpretive theories: pure forecasting theory, preferred habitat theory, liquidity preference theory and market fragmentation theory. Since 1930, J. Fisher highlighted the role of expectations in determining the nominal interest rate level. The effect of Fisher (1930) is defined as the increase in the nominal interest rate resulting from the effects of these expectations. Hertz (1940) asked for Fisher's idea of anticipation, and then in the uncertain future with Meiselman (1962). It regards clients as speculators indifferent to capital risks. Modigliani and Soch (1966) are analyzed as extensions of the previous two theories. Brokers naturally do not prefer a short horizon for lenders and a long horizon for exporters, but they determine the horizon that determines the preferred habitat of each operator who wants to protect it entirely from

interest rate risk by adopting an equation of the duration of instruments to their investment horizon as a single investment base. This horizon depends mainly on the structure of its resources. Thus, can be analyzed the structure of the curve through the price to pay for any stakeholder to agree to abandon their preferred habitat. It therefore appears as a premium for imbalance in the rate structure. It can be positive or negative. On the other, the theory of preference for liquidity, if the issuers of securities have different preferences than investors, for example, if issuers prefer long-term securities while investors prefer short-term securities, then issuers will have to offer a premium for the investors to renounce their initial choice. This bonus is called the liquidity premium. On the other, the theory of preference for liquidity, if the issuers of securities have different preferences than investors, for example, if issuers prefer long-term securities while investors prefer short-term securities, then issuers will have to offer a premium for the investors to renounce their initial choice. This bonus is called the liquidity premium. This situation is then considered the most common, and the longer the loan lasts, the higher the non-liquidity premium. Culbertson (1957) was developed the theory of the rate structure, who emphasizes the fact that investors have preferences for certain maturities. These researchers have been trying to determine the mechanisms affecting the shape and evolution of the structure by term. Various models have been proposed to describe the term structure of interest rates. These models are used to describe the relationship between the rate and its term. Besides, all financial market practitioners are interested in understanding the techniques for plotting the interest rate curve. Much of the financial literature has been devoted to the study of the term structure of the interest rate. The purpose of the equilibrium model is to determine the term structure of interest rates. This type of model is based on a single variable. The models of this theory are equilibrium models such as the Vasicek (1977) models, Cox, Ingersoll and Ross (1985) are two models with a single variable that determines the evolution of the interest rate. Next, the two models of Salomon Brothers and the "paris-bas" model, based on the assumptions that the entire structure per term at any time can be expressed in terms of the returns of the instruments without defects of the longest and shortest maturity. In a comparison with a yield curve model previously proposed by Cohen et al.

### 3. Methodology

In concrete terms, these bonds have created more liquidity. So, the yield bond is based on a number of variables that are used in the measurement. The analysis of the variables makes it possible to measure the actuarial rate of return, maturity and coupon. In this context, there are some models econometric is based on actuarial rates of return. The basic idea is to find a relationship that explains the rate of return of a security according to its maturity and coupon level, the relationship is written as follows:

$$Y^2 = f(m^3, c^4)$$

$Y$ : This is the actuarial rate of return on the bond

$m$ : It's maturity

$c$ : Coupon that implicitly reflects the change in the interest rate

We can cite, as an example, the model of Salomon Brothers tested for the first time in 1976 on a bond portfolio, this model relates the actuarial yield, the maturity and the coupon. The model Salomon Brothers:  $Y = a_0 +$

$$a_1 \log(m) + a_2 \frac{1}{m} + a_3 \times C$$

## 4. Results and Discussion

### 4.1 Data and descriptive static

In this paper, we analyze different variables, Actuarial rate of return, maturity, coupon. More precisely, we examine these variables during the period from January 1, 2010 until December 31, 2022 on annual frequencies and all financial information (central bank financial data) relating to bond issues of financial and banking institutions in Tunisia. So, we present the descriptive static of the endogenous variables and the explanatory variables which appear in the following Table1:

**Table 1:** descriptive static of different variables

	ARR	M	C	1/M	Log(M)
<b>Mean</b>	0.057	4.99	2.81	0.33	0.6

<b>Std.dev</b>	0.012	3.057	1.9	0.28	0.31
<b>Max</b>	0.1	13	8.54	1	1.114
<b>Min</b>	0.0365	1	0.55	0.077	0
<b>Skewness</b>	1.18	0.6	1.04	1.54	-0.52
<b>Kurtosis</b>	4.01	2.57	3.62	4.16	2.36
<b>p-value</b>	0.000	0.000	0.000	0.000	0.000

From the Table 1, Actuarial rate of return =0.012 whereas the lower risk. But, other variable to have a high standard deviation (Maturity= 3.057). The asymmetry between different variables in terms of skewness and kurtosis are well documented, implying they are normally distributed.

## 4.2 Correlation matrix

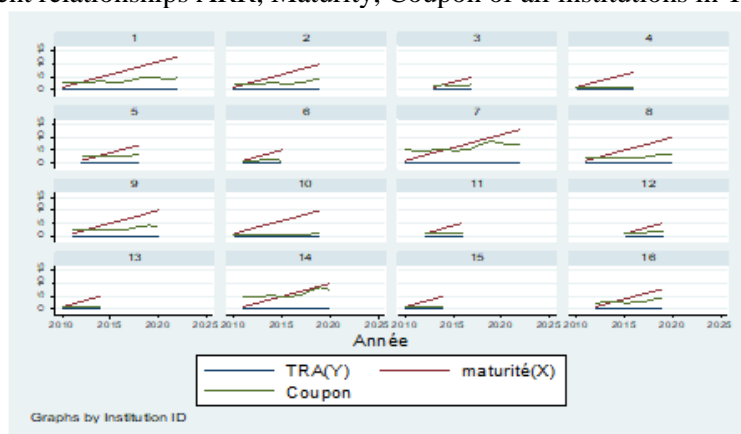
**Table 2:** Correlation matrix of different variables

	ARR	M	C	1/M	Log(M)
<b>ARR</b>	1				
<b>M</b>	0.82	1			
<b>C</b>	1	0.82	1		
<b>1/M</b>	-0.5	-0.77	-0.5	1	
<b>Log(M)</b>	0.71	0.94	0.71	-0.93	1

Based on the result of the correlation matrix, the relationship between the actuarial rate of return and maturity, coupon, interest rate is a relationship positive.

## 5. Analysis Graphic

We show that the different relationships ARR, Maturity, Coupon of all institutions in Tunisia.



**Figure 1:** Plots of actuarial rate of return, maturity and coupon

It can say, this Figure N°1 the variable ARR (TRA (Y)) represented in parallel with Coupon and Interest rate. But, we compare difference lower between ARR-coupon. Besides, the relationship between ARR and Maturity are starting from the same position, but the curve of maturity increases remarkably.

## 6. Empirical validation

### 6.1. Results from unit root test

**Table 3:** Unit root test of different variables

TypeTest	Levin-Lin-Chu		Fisher-type Test		
	Test				
Variable	ARR	Coupon	M	Log(M)	1/M

t-statistics	-1.39	-1.39	P	18.52	232.32	562.9
						4
			Z	0.16	-5.76	-17.53
			L*	0.14	-14.7	-38.83
			Pm	-1.68	25.04	66.36

We first examine these different variables using two classical unit root tests: «Levin-Lin-Chu (2002) test» and Fisher-type unit-root test. I'm performing several tests on panel data. From Table 3, we apply a unit root test with different test such as the augmented «Levin-Lin-Chu» test. In level, the t-statistics of these variables are greater than the critical values of «Levin-Lin-Chu» test. We can accept the hypothesis (H1) is said different variable ARR, Coupon to be stationary and statistical significance. But, the variable «Maturity» is tested by Fisher-type unit root test:

$H_0$ : All panels contain unit roots

$H_1$ : At least one panel is stationary

If you look at your tests P,Z,L\* and Pm, the null hypothesis of this test is that all panels contain a unit root. Given your results we reject hypothesis null. If you look at your tests P, Z, L\* and Pm, you get a value for these test statistics of variable Maturity (18.52; 0.16 ; 0.14 and -1.68). But, we used Logarithmic of variable Maturity for change result by test Fisher unit root. This new result shows that variable becomes stationary “t-statistic=0.00”.

## 6.2. Estimation results using different methods

The results obtained show the estimation of a relationship between the actuarial rate of return, maturity and coupon.

**Table 4:**

actuarial rate of and coupon		Model0		Model1			Estimation of return, maturity
		Maturity	Coupon	Log(M)	1/M	Coupon	
<b>Methods1:</b>	<b>coeff</b>	0.0022	-0.00004	0.05	0.037	-0.0004	
	<b>OLS</b>						
	<b>Std.dev</b>	0.00036	0.0005	0.01	0.01	0.0005	
	<b>P&gt; t</b>	0.000	0.942	0.000	0.001	0.939	
	<b>F</b>	25.6		17.1			
	<b>P&gt;F</b>	0.000		0.000			
<b>Methods2:</b>	<b>R<sup>2</sup></b>	0.29		0.292			
	<b>coeff</b>	0.0009	0.01	0.02	0.013	0.01	
	<b>Perform</b>						
	<b>Std.dev</b>	0.0002	0.0009	0.006	0.005	0.0009	
	<b>FE-</b>						
	<b>P&gt; t</b>	0.000		0.001	0.021	0.000	
<b>and RE-</b>	<b>F</b>	238.85		159.97			
	<b>P&gt;F</b>	0.000		0.000			
	<b>coeff</b>	0.0016	0.006	0.034	0.025	0.007	
	<b>Std.dev</b>	0.0002	0.0009	0.006	0.006	0.0009	
	<b>P&gt; t</b>	0.000		0.000			
	<b>chi2</b>	280.54		290.14			
<b>model</b>	<b>P&gt;chi2</b>	0.000		0.000			

We use a double step method to estimate from table4 the long a term relationship between different variables. Given some drawbacks of using such method, one might use the « OLS » and «Perform FE- and RE-model».

### a. OLS method

$$\text{Model 0: } Y = f(m, c)$$

Such relationship accepted ex post under the stationary in the level of the residuals of long term relationship, we show this variable maturity has to impact positively on the ARR whereas coupon does not influence, This indicates that it is not statistically significant  $p = 0.942 > 0.05$ .

$$\text{Model 1 "Salomon Brothers": } Y = a_0 + a_1 \log(m) + a_2 + a_3 \times C$$

From the estimation result, we notice that this model is significant ( $p > F(17.1) = 0.000$ ). We used logarithmic of maturity affected ( $\log(\text{Maturity})$ ) positively and significantly influences the ARR. Besides, Inverse Maturity has an impact positively on the ARR. But, whereas coupon does not influence it.

**b. "Perform FE- and RE-model" method:**

$$\text{Model 0: } Y = f(m, c)$$

Such relationship accepted ex post under the stationary in the level of the residuals of long term relationship, we show this variable maturity ( $p = 0.001 > 0.05$ ) has to impact positively on the ARR whereas coupon does influence, which indicates that it is statistically significant  $p = 0.000 > 0.05$ .

$$\text{Model 1 "Salomon Brothers": } Y = a_0 + a_1 \log(m) + a_2 + a_3 \times C$$

From the estimation result by "Perform FE (Unbalanced)- and RE-model (balanced)", we notice that this model is significant ( $p > F = 0.000$ ). We used logarithmic of maturity affected ( $\log(\text{Maturity})$ ) positively and significantly influences the ARR. Besides, Inverse Maturity has an impact positively on the ARR. On the other hand, the variable coupon gives the same result does influence it.

## 7. Conclusion

In this paper, we attempt to investigate the association between ARR, maturity and coupon over the period 01/01/2010-31/12/2022. In this regard, the result of studying yield bonds of corporate data suggests that there are some problems in developing a data set that is somewhat homogeneous. Nevertheless, the empirical models tested in this paper performed of these models, proves to provide a high level of explained variation in the yields observed in the Tunisian bond market. The models that are reported in the yield bonds literature as being good explanatory models were run with corporate. These Models Salomon Brothers are significant and have the expectations theory of the term structure as their theoretical base. Predicted yields bonds, coupon and maturity were calculated for a holdout sample and found to be accurate. The estimation of our models applied to financial institutions over a period of 13 years, we show that: -Maturity is significant and gives a positive relationship with the actuarial rate of return.

-The coupon is significant but gives a positive relationship with the actuarial rate of return. From these results, we found that the interest rate has an impact on the bond yield. These results indicate that useful yield bonds can be formed from ex post corporate bond data. These yield bonds can be employed in an ex ante sense to obtain bond prices, a host of empirical questions can be investigated.

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