Estimation of Distribution Function Parameters for Cases of Risk of Mortality Rate due to Malnutrition and Unhealthy Sanitation in Indonesia

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Abstract

Child undernutrition is a significant problem in Indonesia; persistently high rates of stunting, underweight and wasting. Data about malnutrition and sanitation that taken for this research is data of age-standardized death rate, measured per 100,000 individuals from unsafe sanitation and malnutrition in Indonesia. The purpose of this research is to determine the distribution function and estimate the parameter distribution, so the values can provide identification of risk events. The method used for this research is Maximum Likelihood Estimation (MLE) and Newton Raphson iterations. The distribution function formed is gamma and Generalized Pareto Distribution (GPD), respectively for sanitation and malnutrition problems in Indonesia. The projected probability of occurrence of the risk of death due to malnutrition tends to be low in the future. So that the risk classification of the mortality rate due to malnutrition is considered low based on the results of the probability distribution approach on the GPD function. While, the projected probability of occurrence of the risk of death due to sanitation tends to decrease in the future. Based on the graph, the risk value with a high probability is around 20. So, the risk classification of the mortality rate due to malnutrition is considered moderate based on the results of the probability distribution approach on this gamma function.

Keywords: Malnutrition, Sanitation, MLE, Newton Raphson, Gamma, Generalized Pareto Distribution

1. Introduction

Child undernutrition is a significant problem in Indonesia; persistently high rates of stunting, underweight and wasting affect children under five years of age. Stunting reflects chronic undernutrition, which can have severe long-term consequences including: stunted growth, diminished cognitive and mental ability, susceptibility to disease, low economic productivity and poor reproductive outcomes. Wasting results from acute deprivation of nutrition and frequent illness, and significantly increases the risk of child mortality. Both conditions stem from inadequate and improper nutrition during all stages of a child's life and can have significant implications for children's long-term health and survival, and thus for the country's economic productivity and ability to achieve national and international development goals (Maehara, et al. 2019; Mahmudiono, et al. 2017).

Adequate and equitable access to quality water, sanitation and hygiene (WASH) is crucial for preventing disease and ensuring good health, nutrition and development outcomes for children. Indonesia has made considerable progress in increasing access to improved water supply and sanitation across the country and has shown strong commitment to improving access through the promotion of community-based total sanitation (known as STBM in Indonesia) and other sanitation programs. Nonetheless, when the issue of 'safely managed' services is considered (as in SDG targets) – implying treatment of fecal waste and drinking water – success rates drop dramatically. The Government of Indonesia defines 'access to safely managed sanitation' to include households with access to a private improved sanitation facility either connected to a sewerage system or septic tanks and reported desludging over the last 5 years. Access is currently just 7 per cent nationally; access to safely managed drinking water is also estimated to be very low. Access to WASH services varies both by geographic location and socio-economic group (Mboi, et al. 2018; Sandjada, et al. 2013).

Research carried out in Indonesia in 2018 found that 29.9 per cent of children younger than 24 months of age experienced some form of stunting, a decline from previous years but still well above the regional average (22 per cent). This research also found that 30.8 per cent of children under five were stunted – a decline from the 37 percent

prevalence estimated in 2013. 48 Significant regional differences were identified; childhood stunting was most prevalent in the country's west and far east and more widespread in rural than urban areas. A subsequent study found that in some locations stunting rates among children run as high as 42 percent (Oddo, et al. 2019; Beal, et al. 2018).

Indonesia's incidence of childhood wasting, which greatly increases the risk of death and illness, is the fourth highest in the world, affecting more than 10 per cent of children under five (over 2 million). Wasting is more common in rural areas. It can be seen in Figure 1 that a map of the presence of stunting in Indonesian regions (Watson, 2019).

Figure 1. Stunting Prevalence in Indonesia

Chronic public under-investment in sanitation has led to underperforming sanitation services. Current WASH expenditure in Indonesia is only US$3 per capita, or 0.08 per cent of total Indonesian GDP. Meanwhile, the total investment required to achieve the sanitation target set by the medium-term development plan is around US$9.2 billion; approximately US$36 per capita. Today about three quarters of Indonesians have access to at least basic sanitation facilities, with substantial disparities between rural and urban areas. Sharp differences also exist among provinces: 59.9 per cent of households have access in DKI Jakarta, compared to only 30.1 per cent in Papua (WHO, 2019).

Diarrhoea – primarily due to poor water, sanitation, and hygiene – is a major cause of death of Indonesian children under five years of age. The majority of Indonesians (89 per cent) have access to at least basic drinking water services (i.e., improved drinking source within a 30-minute roundtrip collection time). Concern is growing, however, about water quality. A recent UNICEF-Government study found widespread faecal contamination in 89 per cent of drinking water sources, even in one of the best-performing provinces (Yogyakarta, the first province to achieve open defecation-free status).58 The report also found significant disparities between urban and rural areas and based on wealth. Evidence of leaking from on-site sanitation also underscores the need for safely managed sanitation services. Thus a wide gap remains to be filled between the current reality and the SDG ambition of universal, equitable access to all elements of WASH by 2030 (Cronin, et al. 2017; Luby, et al. 2018).

The practice of open defecation also poses a serious threat to public health and the environment. According to 2018 data from Indonesia's national socio-economic survey (Susenas), the national prevalence of open defecation stood at 9 per cent in 2018, meaning that more than 20 million people still engaged in the practice. This puts Indonesia among the top three countries in the world for open defecation. The problem is particularly acute in rural areas, where close to half the population has been reported to still practice open defecation. Achieving open defecation-free communities is crucial to optimizing gains in child health and nutrition. It can be seen in Figure 2 that a map of the presence of practicing open defecation in Indonesian regions (WHO, 2019; UNCF, 2020).

Figure 2. Practicing Open Defacation in Indonesia

The risk of death due to malnutrition and unhealthy sanitation can be seen as a probabilistic event, because the risk event can occur due to unexpected effects. Examples of unforeseen effects include a pandemic that causes an economic crisis, it can make malnutrition rates increase, or it becomes difficult to get good sanitation facilities. One way to find out the characteristics of the risk with the probability of its occurrence is to apply the distribution function which is estimated based on the historical data of the risk. Determination of distribution function and estimation of parameter distribution values can provide identification of risk events.

Based on the description, the research proposed in this paper is estimate the distribution function parameters for cases of risk of mortality rate due to malnutrition and unhealthy sanitation in Indonesia.

2. Materials and Methods

2.1. Materials

The object in this paper uses data about malnutrition and bad sanitation in Indonesia on 28th years period (1990-2017). Data about malnutrition and sanitation that taken for this research is data of age-standardized death rate, measured per 100,000 individuals from unsafe sanitation and malnutrition in Indonesia.

2.2. Methods

The methods used in this paper are Maximum Likelihood Estimation (MLE) and Newton Raphson Iteration for parameters estimation, then Kolmogorov Smirnov, Anderson Darling and Chi Squared for distributin test.

2.2.1. Maximum Likelihood Estimation (MLE)

Maximum Likelihood Estimation (MLE) is an estimation method that maximizes the function likelihood to get the parameter estimate. The likelihood function is obtained from multiplying probability random sample density function. The function equation likelihood as follows (Smith, 1985):

$$ln\, L(\xi, \sigma \mid x_1, x_2, \ldots, x_n) = ln \prod_{i=1}^{n} f(\xi, \sigma; x_i) \tag{1}$$

The estimated value is obtained when the derivative equation first forms a *closed form* equation. If equations that are formed are not closed form, then further numerical analysis is required for its completion.

2.2.2. Newton Raphson Iteration

One of the numerical analysis used for solve equations that are not *closed form* is Newton Raphson. If $G(\theta)$ is a vector of first derivative of $\ln L(\xi; \beta; x)$ and $H(\theta)$ is matrix of the second derivative of the ln *likelihood* function. also called the Hessian matrix. General equation Newton Raphson obtained from the derivation Taylor series as follows (Mathews and Fink, 1992):

$$\theta_{l+1} = \theta_l - g(\theta_l) H^{-1}(\theta_l) \tag{2}$$

Iteration stops when $\varepsilon_t = \left|\hat{\theta}_t - \hat{\theta}_{t-1}\right| < \varepsilon$.

2.2.3. Goodness of Fit

Goodness of fit can determined by the Kolmogorov-Smirnov test. This test is calculated by adjusting the sample distribution function (empirical) with certain theoretical distributions. According to Frank and Massey (1951) to get conclusion then compare $D_{calculate}$ and $D_{1-\alpha}$ in Kolmogorov-Smirnov table with significance level (α). Hyphothesis be accepted if $D_{calculate} < D_{1-\alpha}$.

In this reasearch goodness of fit process for *GPD* on extreme data from the above of threshold used *EasyFit* software. Package or command that used is Tools Goodness of Fit to get the result of Kolmogorov-Smirnov test

2.2.4. Generalized Pareto Distribution (GPD)

Generalized Pareto Distribution (GPD) is defined as the distribution limit of scaled excesses above the threshold value. Suppose x is a random variable from loss risk with 2 GPD parameters, the GPD distribution function of *X* as follows (Kang and Song, 2017)

$$g_{\xi,\beta}(x) = \begin{cases} \frac{1}{\beta}\left(1+\frac{\xi}{\beta}x\right)^{-1-\frac{1}{\xi}}, \xi \neq 0 \\ \\ \frac{1}{\beta}\exp\left(-\frac{x}{\beta}\right), \xi = 0 \end{cases} \tag{3}$$

where $\beta > 0$; $x \geq 0$ if $\xi > 0$; $0 \leq x \leq -\frac{\beta}{\xi}$ if $\xi < 0$ with ξ: shape parameter and β: scale parameter.

The GPD can be divided into three types based on the shape parameter value (ξ), there exponential distribution if the value $\xi = 0$, pareto type I distribution if $\xi > 0$, and pareto type II distribution if $\xi < 0$.

2.2.5. Estimation of GPD Parameter

F Likelihood function for $\xi = 0$ of probability density function GPD is

$$L(\beta|x_1,x_2,\dots,x_n) = \beta^{-n}e^{-\sum_{i=1}^{n}\frac{x_i}{\beta}}$$

ln likelihood function for $\xi = 0$ of probability density function GPD is

$$\ln L(\beta|x_1,x_2,\dots,x_n) = -n\ln\beta - \frac{1}{\beta}\sum_{i=1}^{n}x_i$$

Estimated scale parameter β is obtained by making the first derivative of the ln likelihood function equal to zero, so formula of scale parameter estimation is (Jockovic, 2012)

$$\hat{\beta} = \frac{1}{n}\sum_{i=1}^{n}x_i \tag{4}$$

where β : scale parameter, n : lots of extreme data, and x_i : extreme data on index-i

The likelihood function of the GPD probability density for is given

$$L(\xi,\beta \mid x_1,x_2,\dots,x_n) = \beta^{-n}\prod_{i=1}^{n}\left(1+\frac{\xi x_i}{\beta}\right)^{-\left(\frac{1}{\xi}+1\right)}$$

The ln likelihood function of equation 2 is as follows.

$$\ln L(\xi,\beta \mid x_1,x_2,\dots,x_n) = -n\ln\beta - \left(\frac{1}{\xi}+1\right)\sum_{i=1}^{n}\ln\left(1+\frac{\xi x_i}{\beta}\right)$$

The next step after getting the ln likelihood function is to get the first derivative of the parameter ξ

$$\frac{\partial \ln L}{\partial \xi} = \frac{1}{\xi^2}\sum_{i=1}^{n}\ln\left(1+\frac{\xi x_i}{\beta}\right) - \left(\frac{1}{\xi}+1\right)\sum_{i=1}^{n}\frac{x_i}{(\beta+\xi x_i)}$$

$$\frac{\partial \ln L}{\partial \beta} = \beta^{-1}\sum_{i=1}^{n}\ln\left(-n+(1+\xi\frac{\xi x_i}{\beta})\right) - \left(\frac{1}{\xi}+1\right)\sum_{i=1}^{n}\frac{x_i}{(\beta+\xi x_i)}$$

Then make the first derivative equation equal to zero until a closed form equation is formed to get the parameter estimation as follows

$$\hat{\xi} = \frac{\sum_{i=1}^{n}\ln\left(1+\frac{\hat{\xi}_i}{\hat{\beta}}\right)}{(1+\hat{\xi})\sum_{i=1}^{n}\frac{x_i}{(\hat{\beta}+\hat{\xi}_i)}} \qquad \hat{\beta} = \frac{(1+\hat{\xi}-n\hat{\xi})\sum_{i=1}^{n}x_i}{n^2}$$

Equation 4 is an equation that is not closed form because there are still parameters in the final equation. One of the solutions for equations that are not closed form is the Newton Raphson method. The use of the Newton Raphson method is done by iterating until a convergent result is obtained. Newton Raphson's general equation is as follows.

$$\theta_{l+1} = \theta_l - g(\theta_l)H^{-1}(\theta_l)$$

$g(\theta)$ is a gradient vector of size where is the number of parameters. $g(\theta)$ contains the first derivative of the GPD probability density function against its parameter. $H(\theta)$ is thesized Hessian $p \times p - matrix$ containing the second derivative of the parameter.

$$g(\theta) = \begin{bmatrix} \frac{\partial \ln L}{\partial \xi} & \frac{\partial \ln L}{\partial \beta} \end{bmatrix}$$

$$H(\theta) = \begin{bmatrix} \frac{\partial^2 \ln L}{\partial \xi^2} & \frac{\partial^2 \ln L}{\partial \xi \partial \beta} \\ \frac{\partial^2 \ln L}{\partial \xi \partial \beta} & \frac{\partial^2 \ln L}{\partial \beta^2} \end{bmatrix}$$

The second derivative of the ln likelihood function is as follows

$$\frac{\partial^2 \ln L}{\partial \xi^2} = 2\xi^{-3}\left[\xi\sum_{i=1}^{n}\frac{x_i}{\beta + \xi x_i} - \sum_{i=1}^{n}\ln\left(1 + \frac{\xi x_i}{\beta}\right)\right] + \left(1 + \frac{1}{\xi}\right)\sum_{i=1}^{n}\frac{x_i^2}{(\beta + \xi x_i)^2}$$

$$\frac{\partial^2 \ln L}{\partial \beta^2} = \beta^{-2}\left[n - (1 + \xi)\sum_{i=1}^{n}\frac{x_i(2\beta + \xi x_i)}{(\beta + \xi x_i)^2}\right]$$

$$\frac{\partial^2 \ln L}{\partial \xi \, \partial \beta} = \xi^{-1}\left[(1 + \xi)\sum_{i=1}^{n}\frac{x_i}{(\beta + \xi x_i)^2} - \beta^{-1}\sum_{i=1}^{n}\frac{x_i}{\beta + \xi x_i}\right]$$

Newton Raphson's iteration begins by specifying the value θ_0. θ_0 is a vector whose elements contain $\hat{\xi}_0$ and $\hat{\beta}_0$. Then the initial estimated value is substituted for the gradient vector and Hessian matrix. The value of $\hat{\beta}_0$ is approximated by the standard deviation of the data, while it is obtained from the substitution of equation 4 for β to equation 3. The result of the substitution is made equal to zero. The initial estimates of the shape parameters are as follows (Mathias, et al. 2007)

$$\hat{\xi}_0 = \frac{n^2 s - \sum_{i=1}^{n} x_i}{\sum_{i=1}^{n} x_i - n\sum_{i=1}^{n} x_i} \tag{5}$$

2.2.6. Gamma Distribution Parameter

The gamma distribution is the 2-parameter frequency distribution $\alpha > 0$ and $\beta > 0$ given by the equation (Thom, 1958)

$$f(x) = \frac{1}{\beta^\alpha \Gamma(\alpha)} x^{\alpha-1} e^{-\frac{x}{\beta}}; \; x \geq 0 \; 0; \; x < 0 \tag{6}$$

If $f(x; \alpha; \beta)$ is an arbitrary frequency function, the likelihood function is defined as

$$L = \prod_{i=1}^{n} f(x_i; \alpha; \beta)$$

where x_i is the value of $to - i$ in n sample.

The maximum likelihood estimate for the gamma distribution is given [18]

$$\hat{\alpha} = \frac{1 + \sqrt{1 + \frac{4}{3}\left(\ln \bar{x} - \frac{1}{n}\sum_{i=1}^{n}\ln x_i\right)}}{4\left(\ln \bar{x} - \frac{1}{n}\sum_{i=1}^{n}\ln x_i\right)} \tag{7}$$

$$\frac{\bar{x}}{\beta} - \hat{\alpha} = 0 \tag{8}$$

where $\bar{x}$ is the average of the values in n sample.

3. Results and Discussion

3.1. Data Characteristics

Characteristics of data on malnutrition and poor sanitation are plotted in histograms. Histogram plots were made to determine the suitability of the data with GPD and Gamma distribution. The histogram plots of malnutrition and poor sanitation data are shown in Figure 3 and Figure 4, respectively.
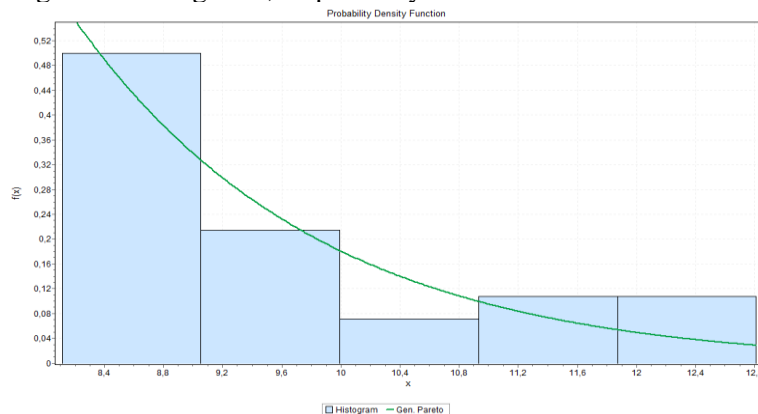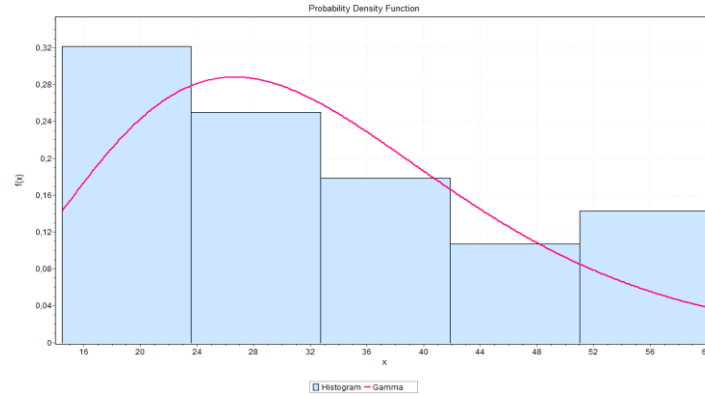
Figure 3. Histogram with Generalized Pareto Distribution Fitting of Malnutrition Data

Figure 4. Histogram with Gamma Distribution Fitting of Poor Sanitation Data

Based on Figure 3, it was found that the malnutrition data matched the distribution of GPD according to the histogram plot. Meanwhile, based on Figure 4, it was found that the histogram plot that is suitable for poor sanitation data is the gamma distribution.

3.2. GPD Parameter Estimation for Malnutrition Data

The initial estimated value $\hat{\beta}_0$ approached by equation (4), so that the initial estimated value $\hat{\beta}_0 = 238,463315$. While the initial estimated value $\hat{\xi}_0$ is obtained as follows.

$$\hat{\xi}_0 = \frac{n^2 s - \sum_{i=1}^n x_i}{\sum_{i=1}^n x_i - n \sum_{i=1}^n x_i}$$

$$\hat{\xi}_0 = \frac{28^2(1,380446209) - 269,383759046}{269,383759046 - 28(269,383759046)}$$

$$\hat{\xi}_0 = \frac{812,88606881}{-7273,361494242}$$

$$\hat{\xi}_0 = -0,111762088$$

Then the initial estimated value is used to perform newton-raphson iterations. The newton-raphson iteration to determine the GPD parameter will stop if the error value is less than the error tolerance ($\varepsilon=0,00001$).

Table 1. Iteration of GPD Parameters Estimation

Number of Iteration	$\hat{\xi}$	$\hat{\beta}$	ε
0	-0.111762	238.463315	
1	-0.111293	238.264117	0.199667
2	-0.110827	238.064962	0.179521
3	-0.110361	237.865846	0.159582
4	-0.109898	237.666767	0.089542
.	.	.	.
.	.	.	.
.	.	.	.
14	-0.105797	235.876127	0.000252

Then the value for the estimated parameter of the Generalized Pareto Distribution function is $\hat{\xi} = -0.105795$ and $\hat{\beta} = 235.876124$. Because the value of $\hat{\xi} < 0$ then this GPD function is included in the type II GPD distribution.

3.3. Gamma Parameter Estimation for Sanitation Data

The sanitation data has $\bar{x} = 32,43568$ so that the estimation of parameters and uses MLE is

$$\hat{\alpha} = \frac{1 + \sqrt{1 + \frac{4}{3}\left(\ln \bar{x} - \frac{1}{n}\sum_{i=1}^n \ln x_i\right)}}{4\left(\ln \bar{x} - \frac{1}{n}\sum_{i=1}^n \ln x_i\right)} = 4,000049$$

$$\hat{\beta} = \frac{\bar{x}}{\hat{\alpha}} = \frac{32,43568}{4,000049} = 8,10882$$

Then the value for the estimated parameter of the Generalized Pareto Distribution function is $\hat{\alpha} = 4.000049$ and $\widehat{\beta} = 8.10882$.

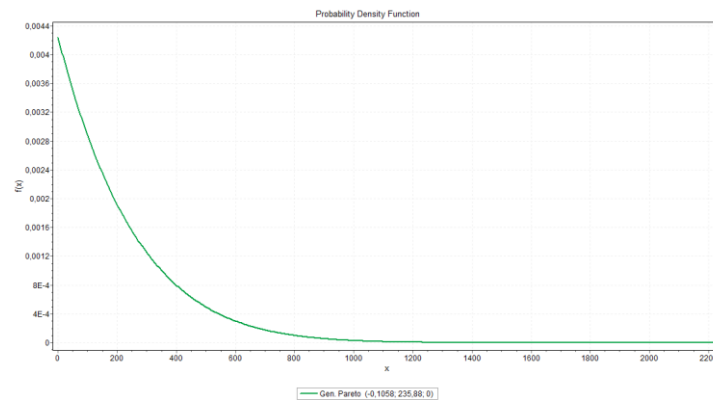### 3.4. Distribution Function Result Analysis

Figure 5. Malnutrition Problem on Generalized Pareto Distribution Graph

Based on the graph above, the projected probability of occurrence of the risk of death due to malnutrition tends to be low in the future. So that the risk classification of the mortality rate due to malnutrition is considered low based on the results of the probability distribution approach on the GPD function. The Results of Kolmogorov-Smirnov Test in Table 2 can be concluded that extreme data values are fit to the *GPD* distribution because there are have no reject on the test. Then GPD parameter can determined from the extreme data values.
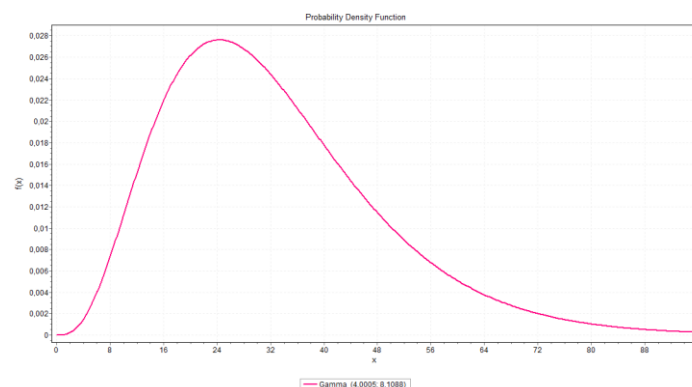

Figure 6. Malnutrition Problem on Generalized Pareto Distribution Graph

Based on the graphic above, the projected probability of occurrence of the risk of death due to sanitation tends to decrease in the future. Based on the graph, the risk value with a high probability is around 20. So, the risk classification of the mortality rate due to malnutrition is considered moderate based on the results of the probability distribution approach on this gamma function.

4. Conclussion

The results of this study have calculated the distribution parameters estimation using MLE and Newton-Raphson iterations. The results obtained for each problem of malnutrition and sanitation are the distribution of GPD and Gamma. The GPD distribution parameter obtained is $\hat{\xi} = -0,105795$ and $\widehat{\beta} = 235.876124$. While the Gamma distribution parameter obtained is $\hat{\alpha} = 4,000049$ and $\widehat{\beta} = 8.10882$.

The projected probability of occurrence of the risk of death due to malnutrition tends to be low in the future. So that the risk classification of the mortality rate due to malnutrition is considered low based on the results of the probability distribution approach on the GPD function. While, the projected probability of occurrence of the risk of death due to sanitation tends to decrease in the future. Based on the graph, the risk value with a high probability is around 20. So, the risk classification of the mortality rate due to malnutrition is considered moderate based on the results of the probability distribution approach on this gamma function.

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