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Implementation of Jackson Queueing Network in General Polyclinic and Dental Polyclinic Patient Services at Jelekong Health Center

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Abstract

The buildup in the service system is generally caused because service facilities cannot meet service needs, so service users need time to use the facilities. Health Center is one of the facilities that provide services in the health sector. However, the number of doctors needed in health centers in Indonesia has yet to be met, and the number of patients who are never empty every day causes a buildup in the service system. The application of the service network system using the open Jackson queueing network can model the optimal service system and analyze its performance. The observation method was used to observe patients' arrival and departure times at the Jelekong Health Center, especially in the registration section, general polyclinic, dental polyclinic, and pharmacy, for three days. There was an increase in patient waiting time before being served and time in the system at the general polyclinic after implementation, so one additional service facility was added. Meanwhile, the dental polyclinic experienced decreased patient waiting time before being served and time in the system, so there was no need to add service facilities.

Keywords: Health center, service system, service network system, Jackson queueing network

1. Introduction

Health has a role in supporting daily activities and is an important factor for human survival. The health that has been given needs to be maintained, so it is necessary to carry out regular checks by visiting health service facilities. However, in 2020, the Central Statistics Agency (BPS) recorded that there were 10,205 inpatient and non-inpatient health centers in Indonesia. The need for adequate doctors does not follow the number of health centers so there is a buildup in the service system.

Stacking problems in the service system can be done by the simulation to estimate the real situation to obtain the analysis results. In everyday life, problems are often encountered for one service system. However, there are problems with more than one interconnected service system in everyday events.

Previous research related to the open Jackson queueing network and service network systems used as references in this research is as follows.

Research conducted by Xia and Shihada (2015) discusses framework optimization on energy and delay optimization problems in multi-hop wireless networks using the Jackson queueing network model. It is proven that in this problem, there is an optimal policy that has a threshold form by formulating the problem as a Markov decision process. Another study was conducted by Choi and Hanaoka (2017) to estimate the reduction of waiting time at the airport during disaster response operations. Open Jackson queueing network estimates the reduced airplane waiting time at each airport. Alam *et al.* (2021) also researched traffic's open Jackson queueing network. The only feasible solution to this problem is to overcome the efficiency problem. The problem is solved by determining when traffic lights turn on from existing data.

Based on the previous description, research was conducted on the health centre's problem of more than one interconnected service system. The health center used for research is Jelekong Health Center, Bandung Regency in the registration section, general polyclinic, dental polyclinic, and pharmacy using the open Jackson queueing network. The research title "Application of Jackson Queueing Network to General Polyclinic and Dental Polyclinic Patient Services at Jelekong Health Center" was obtained.

2. Service System

2.1. Service System Notation

Service systems are a branch of applied mathematics that predicts the behaviour of a waiting line, such as the length of the waiting line over time and the average time spent by something in the line (Joseph, 2020). Taha (2017) denotes the service system as follows.

$$(a/b/c):(d/e/f)$$

where:
a : arrival distribution pattern,
b : service distribution pattern,
c : number of service facilities,
d : service discipline,
e : waiting time capacity, and
f : input source size.

2.1.1. Service System Model $(\boldsymbol{M/M/1}):(\boldsymbol{GD}/\infty/\infty)$

According to Taha (2017), the service system model $(M/M/1):(GD/\infty/\infty)$ based on notation is a service system that has one service facility with no capacity limit and unlimited input source size. The performance of the service system can be calculated with the following equation.

1. Service system effectiveness (ρ).

$$\rho = \frac{\lambda}{\mu} \tag{1}$$

2. Probability of no service users in the system (P_0).

$$P_0 = 1 - \rho \tag{2}$$

3. Average number of service users in the system (L_s).

$$L_s = \frac{\rho}{1-\rho} \tag{3}$$

4. Average service user time in the system (W_s).

$$W_s = \frac{1}{\mu-\lambda} \tag{4}$$

5. Average waiting time for service users to be served (W_q).

$$W_q = W_s - \frac{1}{\mu} \tag{5}$$

6. Average number of service users waiting to be served (L_q).

$$L_q = \frac{\rho^2}{1-\rho} \tag{6}$$

2.1.2. Service System Model $(\boldsymbol{M/M/c}):(\boldsymbol{GD}/\infty/\infty)$

The service system model is almost the same as the service system model $(M/M/1):(GD/\infty/\infty)$ but differs in the number of service facilities as much as c, which means that it can serve a maximum of c service users so that the number of service levels per unit time increases (Taha, 2017). The performance of the equation can be calculated with the following equation.

1. Service system effectiveness (ρ).

$$\rho = \frac{\lambda}{\mu c} \tag{7}$$

2. Probability of no service users in the system (P_0).

$$P_0 = \left[\sum_{n=0}^{c-1} \frac{\rho^n}{n!} + \frac{\rho^c}{c!} \frac{1}{1-\frac{\rho}{c}}\right]^{-1} \tag{8}$$

3. Average number of service users in the system (L_q).

$$L_q = \frac{\rho^{c+1}}{(c-1)!(c-\rho)^2} P_0 \tag{9}$$

4. Average service user time in the system (L_s).

$$L_s = L_q + \rho \tag{10}$$

5. Average waiting time for service users to be served (W_s).

$$W_s = \frac{L_s}{\lambda} \tag{11}$$

6. Average number of service users waiting to be served (W_q).

$$W_q = \frac{L_q}{\lambda} \tag{12}$$

2.2. Open Jackson Queueing Network

Open Jackson queueing network is a network consisting of N service systems (workstations) on a single server. (Zonderland, 2012). According to Sigman (1990), this open Jackson queueing network is interconnected with each workstation, so it is possible to analyze it separately. Faris *et al.* (2020) have conducted research by determining the arrival speed and total service of each workstation depending on the service speed of the previous workstation. The determination of the speed of arrival and total service is written in the following equation.

$$\lambda_j = \alpha_j + \sum_{i=1}^{N} \mu_i P_{ij}, 1 \le i \le N \tag{13}$$

$$\mu_i = \sum_{j=1}^{N} \mu_i P_{ij}, 1 \le j \le N \tag{14}$$

where:
λ_j : total arrival speed to the workstation j,
μ_i : total service speed of the workstation i,
α_j : external speed to the workstation j,
P_{ij} : the probability that a service user moves from a workstation i to j, and
N : number of workstations.

A Jackson transition matrix is formed to determine the probability of moving service users from one workstation to another in the service network system. The matrix is written in the following equation.

$$P = \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1N} \\ P_{21} & P_{22} & \cdots & P_{2N} \\ P_{31} & P_{32} & \cdots & P_{3N} \\ \vdots & \vdots & \vdots & \vdots \\ P_{N1} & P_{N2} & \cdots & P_{NN} \end{bmatrix}, \sum_{j=1}^{N} P_{ij} = 1, 1 \le i \le N \tag{15}$$

After the P value is obtained, the new μ value is determined by forming a matrix multiplication like the following equation.

$$\lambda = \mu P \tag{16}$$

The new service speed is used to analyze the performance of each workstation.

2.3. Goodness of Fit Test

Goodness of Fit Test is a way to determine whether the data is theoretically distributed by comparing two data distributions, namely theoretical and real data. Testing can use the Kolmogorov-Smirnov test by giving a hypothesis. In this study, arrival and service data were tested to follow the Poisson distribution, so the hypothesis is written as follows.

a. H_0 : the data follows a Poisson distribution, and
b. H_1 : the data does not follow the Poisson distribution.

Determining the acceptance or rejection of the hypothesis above is done by calculating the $Asymp - sig(2{\sim}tailed)$ value and comparing it. The significance level is a comparison of the $Asymp - sig(2{\sim}tailed)$ value to get the result of determining the hypothesis which is divided into two possibilities.

a. $Asymp - sig(2{\sim}tailed) >$ Significant level, then H_0 is accepted so the data is Poisson distributed, and
b. $Asymp - sig(2{\sim}tailed) <$ Significant level, then H_0 is rejected so the data is not Poisson distributed.

3. Results and Discussion

3.1. Jelekong Health Center Service Network System

The service network system at the Jelekong Health Center is shown in Figure 1.

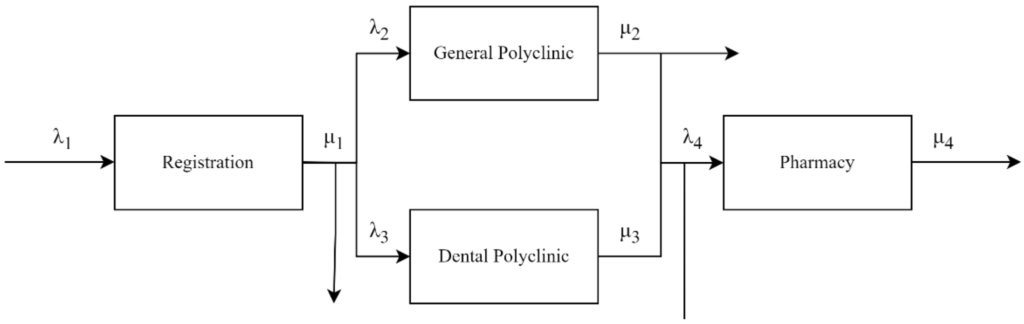

Figure 1: Schematic of Jelekong Health Center service network system

3.2. Steady State Measurement

The observed data is used to determine the arrival and service rates. Then, the arrival and service data are tested for distribution. Next, the performance of the service system is analyzed for each workstation. Finally, an open Jackson queueing network is applied to obtain a new service system performance analysis. Steady state is obtained from the calculation of service effectiveness based on arrival rate and service rate as Table 1.

Table 1: Steady state measurement

Workstation	λ	μ	c	ρ
Registration	0.4278	0.3768	3	0.3785
General polyclinic	0.1352	0.351	1	0.3852
Dental polyclinic	0.0574	0.1281	1	0.4481
Pharmacy	0.3315	0.6462	1	0.513

Based on Table 1, the service effectiveness is < 1 so that each workstation is in steady state. Using the Kolmogorov-Smirnov One-Sample test, the distribution of arrivals and services was tested as Table 2 and Table 3.

Table 2: Arrival distribution test

Null Hypothesis	Test	$Sig.$	Decision
The distribution of K_Pendaftaran is Poisson with mean 26	One-Sample Kolmogorov-Smirnov Test	.054	Retain the null hypothesis
The distribution of K_PoliUmum is Poisson with mean 8	One-Sample Kolmogorov-Smirnov Test	.561	Retain the null hypothesis
The distribution of K_PoliGigi is Poisson	One-Sample Kolmogorov-Smirnov	.756	Retain the null

with mean 3	Test		hypothesis
The distribution of K_Farmasi is Poisson with mean 20	One-Sample Kolmogorov-Smirnov Test	.425	Retain the null hypothesis

Asymptotic significances are displayed. The significance level is 0.50.

Table 3: Service distribution test

Null Hypothesis	Test	$Sig.$	Decision
The distribution of P_Pendaftaran is Poisson with mean 26	One-Sample Kolmogorov-Smirnov Test	.136	Retain the null hypothesis
The distribution of P_PoliUmum is Poisson with mean 8	One-Sample Kolmogorov-Smirnov Test	.824	Retain the null hypothesis
The distribution of P_PoliGigi is Poisson with mean 3	One-Sample Kolmogorov-Smirnov Test	.896	Retain the null hypothesis
The distribution of P_Farmasi is Poisson with mean 15	One-Sample Kolmogorov-Smirnov Test	.131	Retain the null hypothesis

Asymptotic significances are displayed. The significance level is 0.50.

3.3. Service System Performance Analysis

Jelekong Health Center applies the First Come First Serve (FCFS) service discipline for each workstation. All workstations except the Registration workstation have a service system model $(M/M/1):(FCFS/\infty/\infty)$, and the Registration workstation itself has a service system model $(M/M/3):(FCFS/\infty/\infty)$. Explain the service system performance analysis for each workstation as Table 4

Table 4: Service system performance analysis for each workstation

Workstation	P_0	L_q	L_s	W_q	W_s
Registration	0.3411	$0.0815 \approx 1$	$1.2169 \approx 2$	0.1905	2.8845
General polyclinic	0.6148	$0.2413 \approx 1$	$0.6265 \approx 1$	1.7849	4.6339
Dental polyclinic	0.5519	$0.3638 \approx 1$	$0.8119 \approx 1$	6.3379	14.1443
Pharmacy	0.487	$0.5404 \approx 1$	$1.0534 \approx 2$	1.6301	3.1777

3.4. Open Jackson Queueing Network

Equations for the arrival rate and total service were created to obtain the following equations.

$$\lambda_1 = \alpha_1 \tag{17}$$

$$\lambda_2 = \mu_1 P_{12} \tag{18}$$

$$\lambda_3 = \mu_1 P_{13} \tag{19}$$

$$\lambda_4 = \alpha_2 + \mu_2 P_{24} + \mu_3 P_{34} \tag{20}$$

$$\mu_1 = \mu_1 P_{12} + \mu_1 P_{13} + \mu_1 P_{1k} \tag{21}$$

$$\mu_2 = \mu_2 P_{24} + \mu_2 P_{2k} \tag{22}$$

$$\mu_3 = \mu_3 P_{34} \tag{23}$$

$$\mu_4 = \mu_4 P_{4k} \tag{24}$$

The above equation is made into matrix form.

$$\begin{pmatrix}\lambda_2\\ \lambda_3\\ \lambda_4\\ \mu_1\\ \mu_2\\ \mu_3\\ \mu_4\end{pmatrix}=\begin{pmatrix}\mu_1 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & \mu_1 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & \mu_2 & 0 & \mu_3 & 0\\ \mu_1 & \mu_1 & \mu_1 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & \mu_2 & \mu_2 & 0 & 0\\ 0 & 0 & 0 & 0 & 0 & \mu_3 & 0\\ 0 & 0 & 0 & 0 & 0 & 0 & \mu_4\end{pmatrix}\begin{pmatrix}P_{12}\\ P_{13}\\ P_{1k}\\ P_{24}\\ P_{2k}\\ P_{34}\\ P_{4k}\end{pmatrix} \tag{25}$$

The probability value of the patient moving is obtained as follows.

$$\begin{pmatrix}P_{12}\\ P_{13}\\ P_{1k}\\ P_{24}\\ P_{2k}\\ P_{34}\\ P_{4k}\end{pmatrix}=\begin{pmatrix}0{,}3588\\ 0{,}1523\\ 0{,}4889\\ 0{,}5795\\ 0{,}4205\\ 1\\ 1\end{pmatrix}$$

The probability of patient movement is used to determine the new service rate using the matrix.

$$\begin{pmatrix}0 & 0 & 0 & 0\\ P_{12} & 0 & 0 & 0\\ P_{13} & 0 & 0 & 0\\ 0 & P_{24} & P_{34} & 0\end{pmatrix}\begin{pmatrix}\mu_{1b}\\ \mu_{2b}\\ \mu_{3b}\\ \mu_{4b}\end{pmatrix}=\begin{pmatrix}\lambda_1\\ \lambda_2\\ \lambda_3\\ \lambda_4\end{pmatrix}$$

The new service rate is obtained as follows.

$$\begin{pmatrix}\mu_{1b}\\ \mu_{2b}\\ \mu_{3b}\\ \mu_{4b}\end{pmatrix}=\begin{pmatrix}0{,}3768\\ 0{,}1651\\ 0{,}1809\\ 0\end{pmatrix}$$

3.5. Service System Performance Analysis with New Service Rate

The open Jackson queueing network application obtained new service rates for the general polyclinic and dental polyclinic workstations. The service system analysis for the new service rate is shown in Table 5.

Table 5: Service system performance analysis for each workstation with the new service rate

Workstation	ρ	P_0	L_q	L_s	W_q	W_s
Registration	0.3785	0.3411	1	2	0.1905 minutes	2.8845 minutes
General polyclinic	0.8189	0.1811	4	1	27.3879 minutes	33.4448 minutes
Dental polyclinic	0.3173	0.6827	1	1	2.5692 minutes	8.0972 minutes
Pharmacy	0.513	0.487	1	2	1.6301 minutes	3.1777 minutes
General polyclinic ($c = 2$)	0.4094	0.4958	1	2	1.4436 minutes	7.5005 minutes

At the general polyclinic workstation, the average time patients wait to be served and are in the system is getting longer, so one service facility is added to reduce waiting time.

4. Conclusion

Based on the results of the service system analysis and discussion, the conclusions are obtained:

1. The optimal service system model at Jelekong Health Center, Bandung Regency for general polyclinics is $(M/M/2):(FCFS/\infty/\infty)$ and dental polyclinics is $(M/M/1):(FCFS/\infty/\infty)$.
2. The results of the analysis using the open Jackson queueing network at the Jelekong Health Center in general polyclinics found that the average number of patients waiting to be served was 1 patient with an average waiting time of 1,4436 minutes and the average number of patients in the system was 2 patients with time spent for 7,5005 minutes. At the dental polyclinic, it is known that the average number of patients waiting to be served is 1 patient with an average waiting time of 2,5692 minutes and the average number of patients in the system is 1 patient with time spent for 8,0972 minutes.

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