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Mathematical Modelling of Inventory Costs in Supply Chains with Horizontal Cooperation

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Abstract

Efficiency in the supply chain has become very important for companies in the era of increasingly fierce business competition. This has encouraged some companies to start implementing the concept of collaboration between companies or commonly called horizontal cooperation. Horizontal cooperation is important because companies need to work together to manage complex supply chains. To manage production in a collaborative supply chain requires designing a cost-effective model. The design of this model is expected to optimize inventory costs to achieve common goals in horizontal cooperation. This model is a form of linear programme consisting of objective function and constraint function. The objective function is to minimize the inventory cost in a four-tier supply chain with horizontal cooperation on post-production final storage at the production level only. The model was tested with 3 suppliers, 3 materials, 3 factories, 4 products, 3 distributors, and 3 consumers. The model with horizontal cooperation resulted in a total inventory cost of 1,349,830. This amount is 8% lower than the total inventory cost without horizontal cooperation. Thus, horizontal cooperation in the supply chain drastically reduces inventory costs throughout the supply chain.

Keywords: Mathematical model, supply chain, horizontal cooperation

1. Introduction

In the era of globalization and increasingly fierce business competition, efficiency in the supply chain has become crucial for companies to remain competitive (Mota, 2019). However, modern supply chains are increasingly complex due to the involvement of many entities, ranging from raw material suppliers to end consumers. This has encouraged some companies to start implementing the concept of collaboration between companies or commonly called horizontal cooperation. Horizontal cooperation is a form of cooperation by utilising the situation among companies active at the same level of the supply chain to improve performance (Cruijssen, 2007). In this context, horizontal cooperation is important because companies need to work together to manage complex supply chains (Li, 2012).

Inventory cost management is one of the key aspects in the effort to achieve production efficiency (Mpwanya, 2007). In the context of horizontal cooperation, companies cooperate with parallel business partners in the supply chain to increase efficiency, reduce costs, and improve customer service (Simchi-Levi, 2003). Some of the factors that affect inventory costs in horizontal cooperation include delivery times, customer demand levels, inventory policies, order cycles, and various other costs associated with storing, transporting, and managing inventory. To manage production in a collaborative supply chain requires designing a cost-effective model that considers the preparation, production, and post-production processes. The design of this model is expected to optimize inventory costs to achieve common goals in horizontal cooperation. According to Hacardiaux and Tancrez (2022), horizontal cooperation benefits all stages of the supply chain, i.e. there is a significant cost reduction where this collaboration is particularly beneficial for retailers (Hacardiaux, 2022). However, it is still important to understand the existing knowledge about trust relationships in a cooperative environment to avoid undesirable outcomes (Pomponi, 2015).

In addition, modelling inventory costs in horizontal cooperation can also help companies identify opportunities to improve their collaboration and cooperation with business partners (Odeyinka, 2022). By better understanding how inventory costs affect the supply chain in horizontal cooperation, companies can make better decisions, optimize their costs, and provide better services to their customers. Therefore, research on mathematical modelling of inventory costs in horizontal cooperation has great potential to improve the efficiency and competitiveness of enterprises in an increasingly complex business environment.

2. Model Assumptions

The subject matter of this study is to develop and analyze a supply chain system to minimize the total inventory cost and enable horizontal cooperation for product inventory. To describe a supply chain system that fully optimizes inventory costs, a 4-level model is developed. The 4-level supply chain consists of multi-supplier, multi-manufacturer, multi-distributor, and multi-consumer (Perdana, 2023). The distinctive feature of this model is the incorporation of horizontal cooperation at the production level of the supply chain. This helps distributors to be able to select any product at any factory to keep stock available as not all factories produce all products. The objective of this model is to minimize the total inventory cost at suppliers, factories, and distribution in the supply chain with horizontal cooperation.

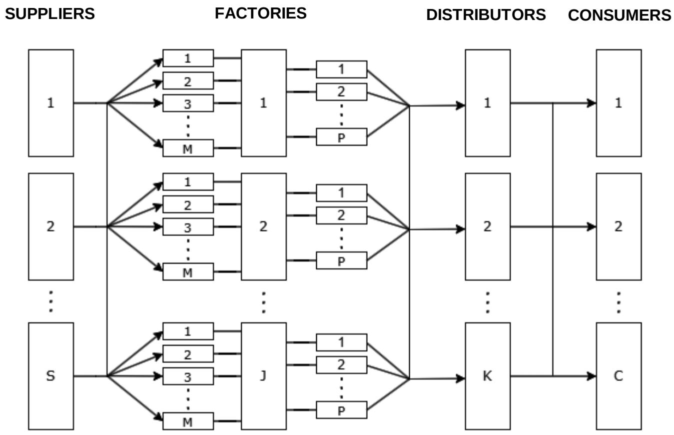

Figure 1: 4-Level Supply Chain

The assumptions built into this model are as follows.

a. Total logistics costs (total logistics costs for supplying materials from suppliers to factories) are included in the inventory costs for this work.
b. Inventory costs include ordering, storage, transport, purchasing, handling, and capital costs.
c. All factories use all types of materials supplied for production.
d. All factories in the supply chain system do not produce all products.
e. All factories must be able to supply all products requested by distributors.
f. Material inventories at suppliers are held at the beginning before supply, product inventories at factories are held at the end after production, and product inventories at distributors are held at the end before delivery to consumers.
g. At the supply stage there is only material storage, the production stage has material and finished product storage, and the distribution stage only has product storage.
h. The inventory infrastructure at the supply level is limited so it is necessary to determine the amount of upper limit in storing the inventory.
i. The base stock level is determined in advance to avoid stock outs.

Figure 1 shows the general structure of a 4-level supply chain consisting of suppliers, manufacturers, distributors, and consumers. It consists of S number of material suppliers with M number of materials they supply at the supply stage. There are J number of factories with P number of products manufactured at the production stage and K number of distributors at the distribution stage. In general, this model has decision variables whose optimal values are to be determined as follows.

Table 1: Variables

Variable	Description
x_{sm}	Number of materials to be supplied
y_{pj}	Number of products to be manufactured
V_{pj}	Amount of products that will not be produced but will be produced by other factories in horizontal cooperation
z_{pjk}	Number of products to be distributed

In this model, the following symbols apply.

Table 2: Symbols

Symbol	Definition	Description
s	Number of suppliers	$s = 1,2, \dots, S$
m	Number of materials	$m = 1,2, \dots, M$
j	Number of factories	$j = 1,2, \dots, J$
p	Number of products	$p = 1,2, \dots, P$
k	Number of distributors	$k = 1,2, \dots, K$
c	Number of consumers	$c = 1,2, \dots, C$

The parameters used in this model are as follows.

Table 3: Parameters

Parameter	Description
$a_{sm}^{(s)}$	Material storage capacity available to suppliers
$C_{sm}^{(s)}$	Material inventory holding cost per unit of product at the supply stage
$h_{sm}^{(s)}$	Material base stock m at the supply stage
$f_{sm}^{(s)} = \begin{cases} 1, & \text{supplier supplies m material} \\ 0, & \text{otherwise} \end{cases}$	Decision parameters at the supply stage
$a_{mj}^{(mp)}$	Available material storage capacity for the factory
$C_{mj}^{(mp)}$	Material inventory storage cost per unit of product at the production stage
$h_{mj}^{(mp)}$	Material base stock m before production at the production stage
r_{mp}	Materials required to produce p units of product
$a_{pj}^{(pp)}$	Product storage capacity available to the factory
$C_{pj}^{(pp)}$	Product inventory holding cost per unit at the production stage
$h_{pj}^{(pp)}$	Base stock of product p after production at the production stage
$\bar{C}_{pj}^{(pp)}$	Inventory holding cost of products not produced by factory j at the production stage
$\bar{h}_{pj}^{(pp)}$	The base stock of products that factory j did not produce after production in the production stage
$f_{mp}^{(mp)} = \begin{cases} 1, & \text{product p requires material type m} \\ 0, & \text{otherwise} \end{cases}$	
$f_{pj}^{(pp)} = \begin{cases} 1, & \text{factory j produces product p} \\ 0, & \text{otherwise} \end{cases}$	Decision parameters at the production stage
$\bar{f}_{pj}^{(pp)} = \begin{cases} 1, & \text{factory j does not produces product p} \\ 0, & \text{otherwise} \end{cases}$	
$a_{pk}^{(d)}$	Product storage capacity available to distributors
$C_{pk}^{(d)}$	Storage cost of product inventory per unit at the distribution stage
$h_{pk}^{(d)}$	Base stock of product p at the distribution stage
D_{pk}	Number of requests for product p by distributors
$f_{pk}^{(d)} = \begin{cases} 1, & \text{distributor k distributes product p} \\ 0, & \text{otherwise} \end{cases}$	Decision parameters at the distribution stage
D_{pc}	The amount of demand for product p by consumers

3. Construction Model

The general form of a linear program consists of two functions, namely the objective functions and the constraint functions. In this model, the objective functions and the constraint functions consists of the following functions.

3.1. The Objective Functions

3.1.1. Function at The Supply Stage

$$F_s(x) = \sum_{s=1}^{S}\sum_{m=1}^{M}\left[C_{sm}^{(s)}\left(h_{sm}^{(s)} + \sum_{j=1}^{J} x_{smj}^{(s)}\right)\right]. \quad (1)$$

The function expresses the sum of all inventory costs for all suppliers and all materials whose costs are calculated individually as the product of the inventory cost and the base stock quantity and the optimal quantity to be supplied by each supplier for each material m and to each factory j.

3.1.2. Function at The Production Stage (Material Storage)

$$F_{mp}(y) = \sum_{j=1}^{J}\sum_{m=1}^{M}\sum_{p=1}^{P} f_{mp}^{(mp)}\, C_{mj}^{(mp)}\, \left[h_{mj}^{(mp)} + r_{mp}\left(y_{pj} + V_{pj}\right)\right]. \quad (2)$$

The function expresses the sum of all inventory costs for all plants and all materials whose costs are calculated individually as the product of the inventory cost and the base stock quantity as well as the optimal product quantity (produced in plant j) and the optimal product quantity (not produced but found in plant j) of each product p of each material m and in each plant j.

3.1.3. Function at The Production Stage (Post-Production Inventory with Horizontal Cooperation)

$$F_{pj} = \sum_{p=1}^{P}\sum_{j=1}^{J}\left[f_{pj}^{(pp)}\, C_{pj}^{(pp)}\, \left(y_{pj} + h_{pj}^{(pp)}\right) + \bar{f}_{pj}^{(pp)}\, \bar{C}_{pj}^{(pp)}\left(V_{pj} + \bar{h}_{pj}^{(pp)}\right)\right]. \quad (3)$$

The function expresses the sum of all inventory costs for all products and all plants whose costs are calculated individually as the product of the inventory cost and the optimal amount that each product p will be produced in each plant j as well as the base stock amount for each product p that is produced in plant j as well as those that are not produced but found in plant j.

3.1.4. Function at The Distribution Stage

$$F_d(z) = \sum_{p=1}^{P}\sum_{k=1}^{K} f_{pk}^{(d)}\, C_{pk}^{(d)}\left(\sum_{j=1}^{J} z_{pjk} + h_{pk}^{(d)}\right). \quad (4)$$

The function describes the sum of all inventory costs for all products and all distributors whose costs are calculated individually as the product of the inventory cost and the optimal quantity that distributor k will distribute produced across plant j as well as the base stock of product p.

3.2. The Constraint Functions

3.2.1. Constraint Function Basic Stock of Ingredients at Suppliers

This constraint function can be expressed as follows

$$c_1 a_{sm}^{(s)} \geq h_{sm}^{(s)} + \sum_{j=1}^{J} x_{smj}^{(s)}, \quad (5)$$

with c_i ensuring the base stock amoun t and the optimal amount of material to be provided from suppliers do not exceed the available storage capacity.

3.2.2. Constraint Function Quantity of Material Supplied

$$\sum_{s=1}^{S} x_{smj}^{(s)} = \sum_{m=1}^{M} r_{mp}\left(y_{pj} + V_{pj}\right). \quad (6)$$

The constraint function states that the optimal amount of material m from supplier s to supply to factory j is equal to the product of the amount of material m required to produce one unit of product p and the optimal amount of product (produced at factory j) and the optimal amount of product (not produced but found at factory j) summed over all types of materials.

3.2.3. Constraint Function of Material Storage Capacity at the Factory

$$a_{mj}^{(mp)} \geq \sum_{p=1}^{P} \left(h_{mj}^{(mp)} + f_{pj}^{(mp)}\, r_{mp}\, y_{pj} \right). \tag{7}$$

The constraint function states that the amount of base stock and the amount of material required to produce a product (for those produced in the factory) must not exceed the product's material storage capacity at the end of factory j, for product p and each material m.

3.2.4. Constraint Function of Product Storage Capacity at Factory

$$a_{pj}^{(pp)} \geq h_{pj}^{(pp)} + f_{pj}^{(pp)}\, y_{pj} + \bar{f}_{pj}^{(pp)}\, V_{pj}. \tag{8}$$

The constraint function states that the base stock quantity, the optimal product quantity (produced in factory j) and the optimal product quantity (not produced but found in factory j) should not exceed the storage capacity of factory j for that product.

3.2.5. Product Demand Constraint Function by Distributor

$$\sum_{j=1}^{J} \left(f_{pj}^{(pp)}\, y_{pj} + \bar{f}_{pj}^{(pp)}\, V_{pj} \right) \geq \sum_{k=1}^{K} D_{pk}. \tag{9}$$

This constraint function states that the number of products produced at the factory and the number of products not produced but produced by other factories for each product p summed over all product types cannot be less than the demand for a product p summed over all distributors.

3.2.6. Constraint Function of Product Storage at Distributor

$$a_{pk}^{(d)} \geq h_{pk}^{(d)} + \sum_{j=1}^{J} z_{pjk}$$

This constraint function specifies that the sum of the base stock at the distribution centre and the optimal amount to be distributed summed over all factories j is not greater than the storage capacity of product p at distribution k.

3.2.7. Constraint Function of Product Demand by Consumers

$$\sum_{j=1}^{J} \sum_{k=1}^{k} z_{pjk} \geq \sum_{c=1}^{C} D_{pc}. \tag{10}$$

This constraint function states that the sum of product demand for all products at each distributor must be more than the sum of all consumer demand.

3.3. Linear Program

Based on the above construction model, the mathematical model of inventory cost in the supply chain with horizontal cooperation is obtained as follows.

Objective Function: minimize the inventory cost of suppliers, factories, and distributors in the supply chain with horizontal cooperation at the final post-production storage at the production level only.

$$\begin{aligned} Min\ F = & \sum_{s=1}^{S} \sum_{m=1}^{M} \left[C_{sm}^{(s)} \left(h_{sm}^{(s)} + \sum_{j=1}^{J} x_{smj}^{(s)} \right) \right] \\ & + \sum_{j=1}^{J} \sum_{m=1}^{M} \sum_{p=1}^{P} f_{mp}^{(mp)}\, C_{mj}^{(mp)} \left[h_{mj}^{(mp)} + r_{mp} \left(y_{pj} + V_{pj} \right) \right] \\ & + \sum_{p=1}^{P} \sum_{j=1}^{J} \left[f_{pj}^{(pp)}\, C_{pj}^{(pp)} \left(y_{pj} + h_{pj}^{(pp)} \right) + \bar{f}_{pj}^{(pp)}\, \bar{C}_{pj}^{(pp)} \left(V_{pj} + \bar{h}_{pj}^{(pp)} \right) \right] \\ & + \sum_{p=1}^{P} \sum_{k=1}^{K} f_{pk}^{(d)}\, C_{pk}^{(d)} \left(\sum_{j=1}^{J} z_{pjk} + h_{pk}^{(d)} \right), \end{aligned} \tag{11}$$

Constraint Function:

$$c_1 a_{sm}^{(s)} - h_{sm}^{(s)} - \sum_{j=1}^{J} x_{smj}^{(s)} \geq 0$$

$$\sum_{s=1}^{S} x_{smj}^{(s)} = \sum_{m=1}^{M} r_{mp} \left(y_{pj} + V_{pj}\right)$$

$$a_{mj}^{(mp)} - \sum_{p=1}^{P} \left(h_{mj}^{(mp)} + f_{pj}^{(mp)} \, r_{mp} \, y_{pj}\right) \geq 0$$

$$a_{pj}^{(pp)} - h_{pj}^{(pp)} - f_{pj}^{(pp)} \, y_{pj} - \bar{f}_{pj}^{(pp)} \, V_{pj} \geq 0$$

$$\sum_{j=1}^{J} \left(f_{pj}^{(pp)} \, y_{pj} + \bar{f}_{pj}^{(pp)} \, V_{pj}\right) - \sum_{k=1}^{K} D_{pk} \geq 0$$

$$a_{pk}^{(d)} - h_{pk}^{(d)} - \sum_{j=1}^{J} z_{pjk} \geq 0$$

$$\sum_{j=1}^{J} \sum_{k=1}^{k} z_{pjk} - \sum_{c=1}^{C} D_{pc} \geq 0$$

4. Simulation

After the formation of the mathematical model as above, the simulation of the mathematical model will be carried out. In this case, the four-level supply chain consists of three suppliers who each supply three materials to three factories where each factory produces four products that are distributed by three distributors to three consumers. It can be written as follows. $S = 3, M = 3, J = 3, P = 4, K = 3$ and $C = 3$. Regarding the value of each parameter, in this simulation it is assumed based on the following data.

Table 4: Company Data Sample With Four-Level Supply Chain

	$C_{sm}^{(s)}$	$a_{sm}^{(s)}$	$h_{sm}^{(s)}$	$C_{mj}^{(mp)}$	$h_{mj}^{(mp)}$	$a_{mj}^{(mp)}$	r_{mp}	$C_{pj}^{(pp)}$	$h_{pj}^{(pp)}$	$a_{pj}^{(pp)}$	$\bar{C}_{pj}^{(pp)}$	$\bar{h}_{pj}^{(pp)}$	$C_{pk}^{(d)}$	$h_{pk}^{(d)}$	$a_{pk}^{(d)}$	D_{pk}	D_{pc}
11	44	130	15	16	15	120	8	40	35	225	20	50	20	15	350	115	30
12	57	120	16	20	18	120	6	35	40	220	10	10	25	20	325	100	45
13	46	100	10	20	19	110	5	30	30	240	15	10	25	25	350	125	15
14	-	-	-	-	-	-	9	20	30	200	30	50	30	30	330	140	20
21	48	180	20	18	20	100	4	45	25	200	20	15	25	25	300	110	30
22	53	150	17	15	25	100	7	20	20	220	40	40	25	20	350	150	40
23	49	200	25	20	15	105	3	30	20	210	40	20	40	40	310	160	50
24	-	-	-	-	-	-	2	40	45	200	30	45	35	20	290	140	10
31	38	125	15	25	12	115	1	35	35	210	45	30	50	45	340	200	50
32	37	135	10	18	10	120	5	35	25	230	50	40	30	25	350	120	10
33	31	100	12	15	20	90	7	40	30	220	25	45	30	30	370	125	15
34	-	-	-	-	-	-	2	45	20	250	30	40	35	40	500	180	50

Table 5: Description Data Sample

Data Sample	Description
$C_{sm}^{(s)}$	Inventory holding cost of material m per supplier s at supply level
$a_{sm}^{(s)}$	Storage capacity of material m per supplier s at supply level
$h_{sm}^{(s)}$	Base stock of material m at the supply stage
$C_{mj}^{(mp)}$	Inventory holding cost of material m per plant j at production stage
$h_{mj}^{(mp)}$	Base stock of material m for each factory j before production at production stage
$a_{mj}^{(mp)}$	Storage capacity of material m per plant j at production stage
r_{mp}	Materials required to produce p units of product
$C_{pj}^{(pp)}$	Inventory holding cost of product p per factory j at production stage
$h_{pj}^{(pp)}$	Base stock of product p for each factory j after production at production stage
$a_{pj}^{(pp)}$	Storage capacity of product p per factory j at production stage
$\bar{C}_{pj}^{(pp)}$	Inventory holding cost of products that factory j does not produce but are found in factory j
$\bar{h}_{pj}^{(pp)}$	Base stock of products not produced by factory j but found in parbik j
$C_{pk}^{(d)}$	Inventory holding cost of product p per distributor k at distribution stage
$h_{pk}^{(d)}$	Base stock of product p for each distributor k at distribution stage
$a_{pk}^{(d)}$	Storage capacity of product p per distributor k at distribution stage
D_{pk}	Demand for product p by each distributor k
D_{pc}	Demand for product p by each consumer c

Simulations were conducted using GAMS Studio software. The process of minimizing the Z function is done using linear program solving. The constraint function that has been formed is developed for each material and its suppliers, products and their producers, as well as distributors and consumers so that 57 constraints are obtained.

5. Results and Discussion

In order to see the full effect of horizontal cooperation on the model, in this case it is studied with and without horizontal cooperation on the end of post-production inventory as modelled. Without horizontal cooperation, the model requires an inventory cost of 1,466,830. Conversely, with horizontal cooperation, this model only requires an inventory cost of 1,349,830 with product 1 not produced but found at factory 1, product 2 not produced but found at factory 2, and product 3 not produced but found at factory 3. These results show that the existence of horizontal cooperation is able to optimize the inventory costs incurred to produce a product. Without horizontal cooperation, each factory is required to produce all semi-finished materials or all products to meet demand. However, with horizontal cooperation, factories do not need to produce all products but can still use the unproduced products. In this case, it is shown that product 1 in factory 1 works better with other factories than producing itself. Similarly, for product 2 in factory 2 and product 3 in factory 3. So, without horizontal cooperation, the total inventory cost in the entire supply chain is higher than with horizontal cooperation in factories.

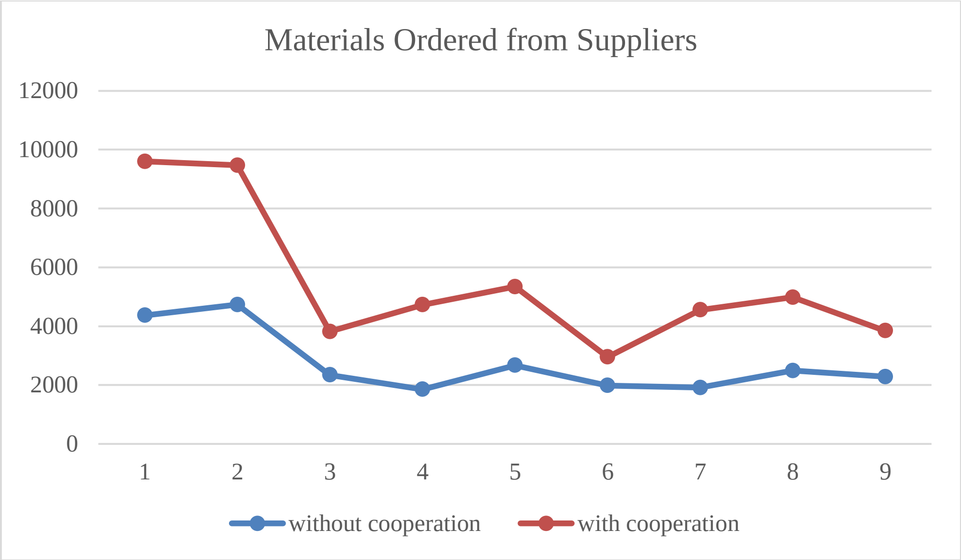

Figure 2: Comparison chart of quantity of materials ordered from suppliers with and without cooperation

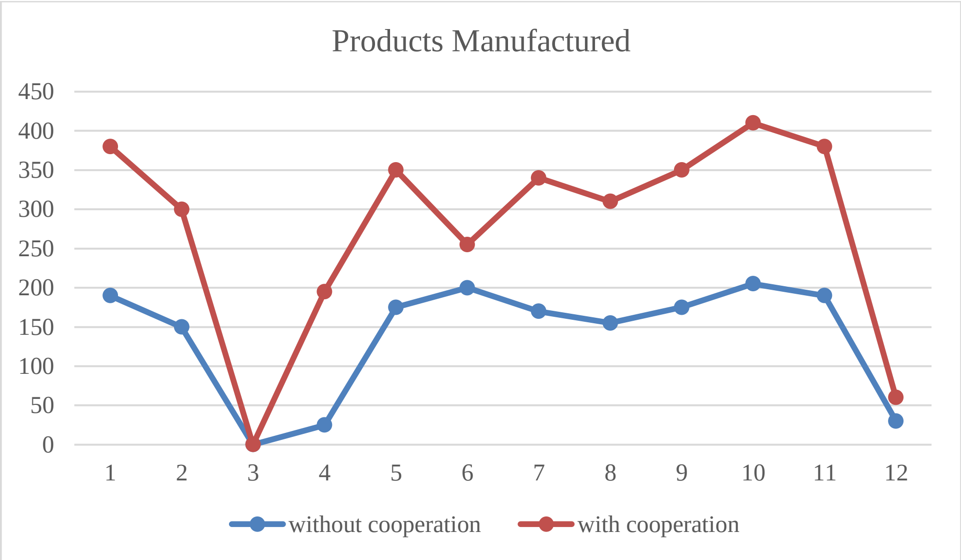

Figure 3: Comparison chart of product quantity produced with and without cooperation

Figure 2 and Figure 3 show graphs comparing materials and products with and without horizontal cooperation in factories. The two graphs clearly show the difference in the amount of materials ordered from each supplier and the amount of products produced in the factories with and without cooperation. Analysis of the results obtained by comparing the models without horizontal cooperation and with horizontal cooperation shows that on average, the materials ordered and the products produced are the same. This is because the goal of this model is to minimize the total cost of inventory so that a significant difference in the total cost of inventory is obtained. With horizontal cooperation, the total procurement cost drops by 8% from the total procurement cost without horizontal cooperation. Thus, it can be concluded that horizontal cooperation in the supply chain drastically reduces inventory costs across the supply chain.

6. Conclussion

The model developed in this study is able to reduce the total inventory cost across the supply chain through horizontal cooperation. This model can be adopted in the supply chain by adjusting the needs of each element. By adopting various forms of cooperation, such as information sharing, role sharing, result sharing, and others, each element in the supply chain can synergize well. Therefore, it can be assumed that horizontal cooperation can improve the performance of all supply chain elements.

However, the performance results with horizontal cooperation still depend on the management and systems implemented in the supply chain. For this reason, further research on mathematical models in the supply chain continues to be necessary. Studies on horizontal and vertical cooperation in supply chains to minimize inventory costs can be conducted in the future. In addition, studies on the continuation of this research, namely optimising decision variables can also be a follow-up to this research.

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