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Balance Analysis of Operational Risk Through the Aggregate Method in the Loss Distribution Approach

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Abstract

Operational risk is defined as the risk of loss resulting from negligence or failure in an entity's internal processes or due to external problems. Companies (especially financial institutions) also face these risks. Recording operational losses in insurance companies is often not done correctly, resulting in limited data regarding operational losses. In this research, the focus is given to operational loss data recorded from claim payments. In general, the number of insurance claims can be resolved using a Poisson distribution, where the expected value of a claim is proportional to its variance. On the other hand, the negative binomial distribution has an expected value that is definitely smaller than its variance. The analytical method used to measure potential losses is through a loss distribution approach using the aggregate method. In this method, loss data is categorized into frequency distribution and severity distribution. By performing 10,000 simulations, a total claim loss value is generated, which is the accumulation of individual claims in each simulation. Then from the simulation results, the potential loss value (OpVaR) at a certain level of confidence is determined.

Keywords: Operational risk, OpVaR, aggregate method.

1. Introduction

The financial services sector often faces tough challenges, especially in dealing with regulatory changes that develop along with technological developments (Gomber, 2018). Insurance, as part of the developing financial industry, also faces risks in carrying out its activities (Zheng, 2018). These risks are inherent in every aspect of the company and reflect potential problems that may be encountered (Knechel, 2007).

One risk that is less thoroughly understood than other risks is operational risk (Chen, 2013). This risk can arise due to inadequacies or dysfunction in internal processes, human error, system failure, or external problems that affect company operations. Even though it looks simple, if not managed well this risk will have a big impact. According to BASEL II (international banking regulations) the measure of operational risk (Operational Value at Risk) is abbreviated as OpVaR.

Recording operational losses, especially in insurance companies, is still not implemented properly, which has an impact on limited data regarding losses in operational risks. In this context, observed operational loss data is obtained from claims payments. In general, insurance claims can be modeled using a distribution with characteristics similar to the Poisson distribution, where the expected value of the claim is equal to its variance, and the negative binomial distribution, where the expected value is smaller than the variance.

2. Literature Review

2.1. Risk Models in the Loss Distribution Approach

The form of the risk model in the loss distribution approach is:

For example:
N = the number of claims generated from the policy portfolio at a certain time.
X_i = the size of the $i-th$ claim, $i = 1,2,\ldots,N$.
So the model of total operational risk losses can be written as follows

$$S = X_1 + X_2 + \cdots + X_N \quad (1)$$

This model is often called the collective risk model. In general, model (1) represents the overall claims of the portfolio at a certain time. The random variable N states the number of claims and is closely related to the frequency of claims.

The random variables $X_1, X_2, \ldots, X_N$ state the size of the ith claim. To make the model easier completed, the following assumptions are required

(i) Random variables N and $(X_1, X_2, \ldots, X_N)$ are independent of each other.
(ii) Random variables $X_1, X_2, \ldots, X_N$ are independent of each other.
(iii) Random variables $X_1, X_2, \ldots, X_N$ have the same distribution.

2.2. Distribution of Total Losses

The total distribution of claims S in a certain time period can be obtained from the distribution of the number of claims N and the distribution of the size of individual claims X. Let X be a random variable that states the size of the claim. It is known that the distribution function of X is F_X. If there are N claims then the total amount of claims is $X_1 + X_2 + \cdots + X_N$ and the distribution is expressed as $F_s\ (s)$.

kth moment = $\varphi_k = E[X^k]$.

Moment generating function of X

$$M_X(t) = E[e^{tX}].$$

Moment generating function of N

$$M_N(t) = E[e^{tN}].$$

Moment generating function of S

$$M_S(t) = E[e^{tS}].$$

To determine the expected value and variance of S, the following two theorems are needed:

Theorem 1: Suppose (X, Y) is a two-dimensional random variable, then the expected value of X can be determined through the expected value of X with the following conditions for Y:

$$E[X] = E[E[X|Y]].$$

Theorem 2: Let (X,Y) be a two-dimensional random variable then:

$$Var[X] = Var[E[X|Y]] + E[Var[X|Y]].$$

On the basis of Theorem 1 and Theorem 2 and in relation to the three assumptions used, the expected value of:

$$E[S] = \varphi_1 E[N], \quad (2)$$

and S variety:

$$Var[S] = E[N]Var[X] + \varphi_1{}^2 Var[N]. \quad (3)$$

Moment generating function of S.

$$MS\ (t) = MN(log(MX(t)). \quad (4)$$

Next, to determine the distribution function of S, it can be seen as follows:

$$\begin{aligned} FS\ (s) &= Pr\ Pr\ (S \leq s) \\ &= \textstyle\sum_{n=0}^{\infty} \quad Pr(S \leq s|N = n)Pr(N = n) \\ &= \textstyle\sum_{n=0}^{\infty} \quad Pr(X_1 + X_2 + \cdots + X_n \leq s)\ P(N = n) \end{aligned}$$

According to the convolution operation for collective risk and according to the assumption X_i have the same distribution $\forall i$,

$$F_S(s) = \sum_{n=0}^{\infty} G^{*n} Pr\,(N = n). \quad (5)$$

If the distribution of the size of individual claims is discrete with a probability function $p(x) = Pr(X = x)$ then the distribution of total claims is also discrete, so the probability function of S can be obtained as follows:

$$f_S(s) = \frac{\partial}{\partial s} \sum_{n=0}^{\infty} G^{*n} Pr\,(N = n)$$
$$f_S(s) = \sum_{n=0}^{\infty} G^{*n} Pr\,(N = n), \quad (6)$$

with

$$g^{*n} = g^* g^* \ldots * g$$
$$= Pr(X_1 + X_2 + \cdots + X_n = s)$$

2.3. Process and Properties of Compound Poisson Distribution

1. Compound Poisson Process

In general, the distribution of the random variable N (the number of claims) is a Poisson distribution with a probability mass function

$$P(N = n) = \frac{\lambda^n e^{-\lambda}}{n!},\ n = 0,1,2,\ldots \quad (7)$$

with $\lambda > 0$.

The expected value and variance of the Poisson distribution are respectively

$$E[N] = Var[N] = \lambda.$$

For example, p_1 and p_2 are the expected values and the 2nd moment of X, it can be stated $E[X] = p_1$ and $E[X^2] = p_2$.

If the random variable N (the number of claims) has a Poisson distribution, then the random variable S in equation (1) has a compound Poisson distribution. So, the expected value and variance of the compound Poisson distribution are

$$E[S] = \lambda p_1, \quad (8)$$

and

$$Var[S] = \lambda p_2. \quad (9)$$

Proof (8):

It is known that $S = X_1 + X_2 + \ldots + X_N$, with $X_1, X_2, \ldots$ spread i.i.d and N spread Poisson. It will be proven that $E[S] = \lambda p_1$:

$$\begin{aligned}
E[S] &= E[E(S|N)] \\
&= E[E(\textstyle\sum_{k=1}^{N} X_k|N)] \\
&= \textstyle\sum_{n=0}^{\infty} E[\sum_{k=1}^{N} X_k|N = n]P(N = n) \\
&= \textstyle\sum_{n=0}^{\infty} E[\sum_{k=1}^{n} X_k]P(N = n) \\
&= \textstyle\sum_{n=0}^{\infty} [\sum_{k=1}^{n} E(X_k)]P(N = n) \\
&= \textstyle\sum_{n=0}^{\infty} [np_1]P(N = n) \\
&= p_1 \textstyle\sum_{n=0}^{\infty} n\,P(N = n) \\
&= p_1 E(N) \\
E[S] &= \lambda p_1.
\end{aligned}$$

Proof (9):

It is known that $S = X_1 + X_2 + \ldots + X_N$, with $X_1, X_2, \ldots$ spread i.i.d and N spread Poisson. It will be proven that $Var[S] = p_2$:

$$\begin{aligned}
E[S^2] &= E[E(S^2|N)] \\
&= E[E((\textstyle\sum_{k=1}^{N} X_k)^2|N)] \\
&= \textstyle\sum_{n=0}^{\infty} E[(\sum_{k=1}^{N} X_k)^2|\,N = n]P(N = n) \\
&= \textstyle\sum_{n=0}^{\infty} [(\sum_{k=1}^{n} E({X_k}^2) + \sum_{k=1}^{n} \sum_{k\neq l}^{n} E(X_k)E(X_l)]P(N = n) \\
&= \textstyle\sum_{n=0}^{\infty} [n\,E({X_1}^2) + (n^2 - n)(p_1)^2]P(N = n) \\
&= E[X^2] \textstyle\sum_{n=0}^{\infty} n\,P(N = n) + (p_1)^2 \sum_{n=0}^{\infty} (n^2 - n)P(N = n) \\
E[S^2] &= E(X^2)\lambda + (p_1)^2\{E[N]^2 - \lambda\}.
\end{aligned}$$

$$
\begin{aligned}
Var[S] &= E[S^2] - (E[S])^2 \\
&= (E(X^2)\lambda + (p_1)^2\{E[N]^2 - \lambda\}) - (E[N]p_1)^2 \\
&= E(X^2)\lambda + (p_1)^2\{E[N]^2 - \lambda\} - ({p_1}^2(E[N])^2) \\
&= E(X^2)\lambda + (p_1)^2\{E[N]^2 - \lambda - (E[N])^2\} \\
&= E(X^2)\lambda + (p_1)^2\{E[N]^2 - (E[N])^2 - \lambda\} \\
&= E(X^2)\lambda + (p_1)^2\{Var[N] - \lambda\} \\
&= E(X^2)\lambda + (p_1)^2 \cdot Var[N] - (p_1)^2\lambda \\
&= \lambda(E[X^2] - (p_1)^2) + \lambda(p_1)^2 \\
&= \lambda(p_2 - (p_1)^2) + \lambda(p_1)^2
\end{aligned}
$$
$$Var[S] = \lambda p_2.$$

The moment generating function for the Poisson distribution is:

$$M_N(t) = e^{\lambda(e^t - 1)}$$

By substituting the Poisson moment generating function, the following equation is obtained:

$$
\begin{aligned}
M_s(t) &= E[e^{ts}] \\
&= E[E[e^{ts} | N]] \\
&= E[M_X(t)^N] \\
&= E[e^{N \log M_x(t)}]
\end{aligned}
$$
$$M_S(t) = M_N[\log M_x(t)].$$

So, the moment generating function for the compound Poisson distribution can be written as follows:

$$
\begin{aligned}
M_S(t) &= M_N[\log M_x(t)] \\
M_S(t) &= e^{\lambda(e^{\log M_x(t)} - 1)} \\
M_S(t) &= e^{\lambda(M_X(t) - 1)}.
\end{aligned}
\tag{11}
$$

The Poisson distribution can only be used if the variance value is the same as the expected value. However, if the variance value of the number of losses is greater than the expected value then the distribution used for the random variable N (number of claims) is a negative binomial distribution with a probability mass function.

$$P(N = n) = \binom{n+r-1}{r-1} p^r q^n, n = 1,2, \ldots \tag{12}$$

with

$$r > 0, 0 < p < 1, q = 1 - p.$$

The expected value and variance of the negative binomial distribution are as follows:

$$E[N] = \frac{rq}{p},\ \ Var[N] = \frac{rq}{p^2}.$$

Suppose p_1 and p_2 are the expected value and the 2nd moment of X respectively, it can be stated:

$$E[X] = p_1, \qquad E[X^2] = p_2$$

If the random variable N (the number of claims) has a negative binomial distribution then the random variable S in equation (1) has a negative compound binomial distribution. Thus, the expected value and variance of the compound negative binomial distribution are obtained as follows:

$$E[S] = \frac{rq}{p} p_1, \tag{13}$$

and

$$Var[S] = \frac{rq}{p} p_2 + \frac{rq^2}{p^2} {p_1}^2. \tag{14}$$

The moment generating function for the negative binomial distribution is:

$$M_N(t) = (\frac{p}{1-q^{e^t}})^r. \tag{15}$$

2. Compound Poisson Distribution Properties

The compound Poisson distribution has two properties, namely:

a. If each random variable has a compound Poisson distribution, then the sum of these random variables also has a compound Poisson distribution.

b. If the random variable S is stated:

$S = X_1 N_1 + X_2 N_2 + \ldots + X_m N_m$,

Then the random variable S has a compound Poisson distribution.

3. Materials and Methods

a. Materials

In this paper, the data obtained is hypothetical data from motor vehicle insurance (frequency and amount of claims per day).

b. Methods

i. i. Measurement of Claim Operational Risk

Theoretically there are several steps that must be taken in calculating
claims reserves with a loss distribution approach using the aggregate method.
These steps are as follows

1) Collection of insurance claims data.
2) Grouping insurance claims data based on distribution and severity.
3) Determine the type of frequency distribution and severity distribution.
4) Determine the parameters of the frequency distribution and severity distribution.
5) Simulate the frequency parameters and severity parameters with n = 10,000.
6) Calculate the total loss from claim payments for each n.
7) Sort the severity from largest to smallest.
8) Calculating unexpected loss (OpVaR of insurance claims).
9) Perform steps 5, 6, 7 and 8 100 times to get the average of unexpected loss (potential loss from insurance claims).

The final stage is to calculate the unexpected loss value, by selecting the desired level of confidence, for example 95% or 99%. For 95%, the unexpected loss value is 5%×10,000 (number of simulations) = 500, meaning the 500th data is the unexpected loss value with a 95% confidence level. While the 99% confidence level can be done the same thing, namely the 100th data is the unexpected loss value with a 99% confidence level or it can also be done directly by looking at the aggregate quartile column which has been sorted from the largest (99.99%) to the highest. smallest (0%). In this paper, the data obtained is hypothetical data from motor vehicle insurance (frequency and amount of claims per day). Therefore, steps 1-4 in the claims reserve calculation are not carried out. The next step, generate data with a Poisson spread frequency distribution with parameters 3.7 as much as $n = 10{,}000$ and generate data with an exponential spread severity distribution with parameters 100.1 as much frequency data as has been obtained for each n, where $n \in [1{,}10{,}000]$.

ii. ii. Total Claim Distribution Approach

a. Normal Approach

Based on the central limit theorem, it is necessary to pay attention to the following 2 things:

1. If S has a compound Poisson distribution with parameters λ and distribution function X is F_x then random variable $Z = \frac{S-\lambda E[X]}{\sqrt{\lambda E[X]}}$ will have a standard normal distribution if $\rightarrow \infty$. The two parameters for this normal approach are

$$E[S] = \lambda E[X] = \lambda_{\varphi_1}$$

and

$$Var[S] = \lambda E[X^2] = \lambda_{\varphi_2}$$

2. If S has a compound negative binomial distribution with parameters r, p and the distribution function X i.e $P(x)$ then random variable $Z = \frac{S - r(\frac{q}{p})\varphi_1}{\sqrt{r(\frac{q}{p})\varphi_2 + r(\frac{q^2}{p^2})\varphi_1{}^2}}$, standard normal distribution if r→∞. The two parameters for this normal approach are

$$E[S] = \frac{rq}{p}\varphi_1$$

and

$$Var[S] = \frac{rq}{p}\varphi_2 + \frac{rq^2}{p^2}\varphi_1{}^2.$$

This normal approach would be better to use if the expected number of claims occurring is large or in other words if λ is large for a compound Poisson distribution or if r is large for a compound binomial distribution.

Because the normal distribution is symmetrical, as a result the central moment of the three is equal to zero or can be written as follows $E[(S - E[S])^3] = 0$. However, the distribution of total claims is often asymmetric or skewed, meaning that the central third moment is not zero. Therefore a more general approach is needed for the distribution of total claims. For this second type of approach, a Gamma distribution translation approach is used.

b. Gamma Translation Approach

If $G(x: \alpha, \beta)$ is denoted as a Gamma distribution function with parameters α and β , then

$$G(x: \alpha, \beta) = \int_0^x \frac{\beta^\alpha}{\Gamma(\alpha)} t^{\alpha-1} e^{-\beta t} dt.$$

In the Gamma translation approach, the parameters $\alpha, \beta,$ and x_0 are chosen by equating the first moment central, second moment central and third moment central of S with the corresponding central moments for the Gamma distribution translation. Therefore, the central moment of the translation of the standard Gamma distribution is:

$$E[S] = x_0 + \frac{\alpha}{\beta},$$
$$Var[S] = \frac{\alpha}{\beta^2},$$
$$E[(S - E[S])^3] = \frac{2\alpha}{\beta^3}$$

So we get:

$$\beta = 2\frac{Var[S]}{E[(S - E[S])^3]}$$
$$\alpha = 4\frac{(Var[S])^3}{E[(S - E[S])^3]^2}$$
$$x_0 = E[S] - 2\frac{(Var[S])^2}{E[(S - E[S])^3]}$$

For compound Poisson distribution, the above procedure with $E[S] = \lambda\varphi_1$, $Var[S] = \lambda\varphi_2$ and $E[(S - E[S])]^3 = \lambda\varphi_3$ will produce the following parameters:

$$\alpha = 4\lambda\left(\frac{\varphi_2{}^3}{\varphi_3{}^2}\right).$$
$$\beta = 2\left(\frac{\varphi_2}{\varphi_3}\right).$$
$$x_0 = \lambda\varphi_1 - 2\lambda\left(\frac{\varphi_2{}^3}{\varphi_3}\right).$$

4. Results and Discussion

Table 1: OpVaR values at 99% and 95% confidence levels (results in tens of thousands

No	OpVar 1%	OpVar 5%
1	1179.57	884.5354
2	1168.349	879.2604
3	1213.011	901.2819
4	1237.793	894.3368
5	1192.684	875.2271
6	1179.725	890.2595
7	1188.218	882.4026
8	1226.822	890.0448
9	1195.007	883.7198
10	1214.221	882.5006
11	1189.816	880.665
12	1203.202	885.5937
13	1222.21	890.7483
14	1232.474	910.1382
15	1177.929	872.6123
16	1221.015	884.8501
17	1227.766	891.5545
18	1170.902	877.54
19	1224.003	894.2159
20	1177.923	880.8441
21	1180.832	883.5471
22	1204.792	880.52
23	1221.216	899.9769
24	1207.817	899.5925
25	1204.295	896.8235
26	1183.913	886.8055
27	1191.17	889.5722
28	1228.467	896.3405
29	1187.943	880.6182
30	1205.715	899.7951
31	1201.839	895.5759
32	1199.738	891.8508
33	1221.709	886.914

34	1211.022	907.9345
35	1199.927	892.9434
36	1185.234	890.0127
37	1187.164	883.9179
38	1195.04	880.9543
39	1214.181	895.9373
40	1203.522	888.6066
41	1213.957	911.4763
42	1222.422	895.4111
43	1208.15	890.1353
44	1195.813	887.3214
45	1215.03	891.581
46	1212.828	882.1126
47	1214.922	896.3549
48	1233.946	904.3978
49	1221.695	898.1123
50	1208.075	888.8549
51	1194.935	905.707
52	1210.044	885.6077
53	1201.926	899.5426
54	1202.596	912.2402
55	1196.574	868.687
56	1188.993	879.3129
57	1199.531	887.445
58	1176.336	878.6872
59	1213.119	904.6092
60	1220.926	897.3182
61	1225.44	892.5123
62	1185.417	885.3677
63	1199.336	888.9523
64	1204.964	901.0305
65	1199.361	896.374
66	1254.722	910.9214
67	1243.561	889.6035
68	1202.311	875.0372
69	1165.936	866.7547
70	1217.812	891.7538

71	1212.552	879.9491
72	1213.443	890.924
73	1189.527	888.5262
74	1211.039	897.7229
75	1216.792	890.8233
76	1224.24	885.7001
77	1189.349	874.1484
78	1232.666	894.7878
79	1217.254	881.723
80	1172.345	891.6238
81	1210.35	903.9724
82	1208.486	896.0464
83	1198.613	880.6819
84	1229.491	881.9569
85	1197.104	898.5799
86	1212.241	889.0343
87	1216.21	886.9924
88	1215.623	886.9123
89	1197.621	882.9609
90	1213.361	892.7376
91	1205.714	884.3733
92	1205.714	884.3733
93	1226.738	896.0856
94	1197.84	883.3441
95	1213.06	878.8313
96	1221.839	906.2982
97	1183.92	887.4438
98	1182.508	884.1437
99	1182.483	890.1959
100	1213.982	885.7684

Table 2: Simulation statistics with 100 repetitions

Statistical Value	**OpVaR 1%**	**OpVaR 5%**

Mean	889.8045	1205.41
St dev	9.297943	17.38954
Max	912.2402	1254.722
Min	866.7547	1165.936

The unexpected loss value of motor vehicle insurance claims in the next day using the loss distribution approach with the aggregate method at α=1% or a 99% confidence level is IDR 12,054,100.00. This means that the maximum potential motor vehicle insurance claim that can be tolerated with a 99% confidence level in the next day is IDR 12,054,100.00. In other words, the maximum claim reserve that must be provided by the insurance company to cover motor vehicle insurance claims for the next 1 day is IDR 12,054,100.00.

Unexpected loss motor vehicle insurance claim with α=5% or 95% confidence level in the next day is IDR 8,898,045.00. This means that the potential for motor vehicle insurance claims that can be tolerated at a 95% confidence level in the next day is IDR 8,898,045.00, so the insurance company must provide a reserve for motor vehicle insurance claims in the next day of IDR 8,898,045.00.

5. Conclussion

The statistical characteristics of operational risk in insurance companies are based on understanding of the concept of collective risk as follows:

a). The distribution of total claims from a policy which is a random event can be found using Risk Theory by first determining the shape of the frequency distribution (number of claims) and severity distribution (size of claims).
b). The distribution of total claims can be calculated using the convolution method, normal approach and Gamma translation approach.
c). In general, the number of insurance claims can be modeled using a distribution that has the same properties as the Poisson distribution, where the expected value of the claim is equal to its variance and the negative binomial distribution, where the expected value is smaller than the variance.

Based on the assumption that the frequency distribution data (number of claims) and severity distribution (size of claims) that are generated are respectively distributed Poisson with a parameter of 3.7 and exponential with a parameter of 100.1, then the calculation results obtained are the amount of claim reserves (OpVaR) that must be prepared at the level 99% and 95% confidence are IDR 12,054,100.00 and IDR 8,898,045.00 respectively.

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