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Calculating Insurance Premiums for Stroke Patients Using the Multistate Markov Chain Method

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Abstract

Health insurance premium is one of the important elements in the insurance industry that needs to be calculated correctly so that insurance companies can minimize risks and losses. In this study, insurance premiums for stroke patients are calculated by utilizing the Markov Chain method. This method is used to model the movement of a patient's health condition over time, considering various conditions such as recovery, relapse, or death. Each condition is represented by a state in the Markov Chain model, and the transition between states is calculated based on patient history data and transition probabilities. Based on the modeling results, a more accurate premium estimation is obtained compared to conventional methods, as it is able to consider the dynamics of changing health conditions. This research provides important insights for the insurance industry in risk management as well as more optimal premium calculations for patients with chronic diseases such as stroke.

Keywords: Insurance premium, stroke patient, Markov chain, transition probability, health model.

1. Introduction

Stroke is a serious medical condition that occurs when the blood supply to the brain is interrupted or reduced, causing brain tissue to lack oxygen and nutrients. (Brusca & Albert, 2023) projects a 50% increase in global stroke outcomes by 2050, driven by population aging and inadequate healthcare infrastructure in vulnerable regions. In Indonesia, stroke is a disease with a high prevalence, especially in the elderly and populations with risk factors such as hypertension, diabetes, and unhealthy lifestyles. This high incidence of stroke presents a major challenge for the healthcare sector, including the health insurance industry, in managing the risks and costs of treatment for stroke patients.

Health insurance plays an important role in providing financial protection for people against unexpected health risks. In India, health insurance is increasingly recognized as a vital social security for the rural poor, aiming to reduce high medical costs and improve access to health care (Ravi, 2023). Accurate and fair calculation of insurance premiums is challenging, especially for chronic and high-risk diseases such as stroke. The use of conventional methods is often inadequate because it is unable to capture the dynamics of the patient's health condition that can change, such as partial recovery, relapse, or even death.

To overcome these limitations, the Markov Chain method offers a more appropriate solution in modeling the movement of a patient's health condition over time. Markov chains are a mathematical approach that can be used to predict the transition probability of a patient's health condition, allowing for a more realistic and scalable pre-calculation of insurance premiums based on the likelihood of the patient's future condition (Becker, 2016).

In this study, we apply the Markov Chain method to model the changing health conditions of stroke patients and calculate the corresponding insurance premiums. Through this approach, it is hoped that the insurance industry can better manage risks, while providing optimal financial protection for stroke-prone insureds.

Table 1: Table of Research Update and analysis on the calculation of Insurance Premiums for Stroke Patients Using the Multistate Markov Chain Method.

Authors	**Variables**	**Method**	**Use of Multistate Markov Chains**	**Use of Single Premium**
Debicka et al., 2022	Marriage dependency	Nonhomogeneous Markov Chain	Yes	No
Villacorta et al., 2021	General insurance	Fuzzy Markov Chain	Yes	No
Torres et al., 2021	Futures trading	Markov Chain	Yes	No
Debicka et al., 2019	Lung cancer patients	Multiple state model	Yes	NO

2. Literatur Review

2.1. Insurance

Insurance is a financial protection mechanism that aims to transfer risk from an individual or entity to an insurance company. By paying a premium, policyholders get a guarantee that they will receive compensation in the future if an insured event occurs. The concept of insurance dates back thousands of years, but continues to evolve along with social, economic and technological changes (Habib, 2016).

Insurance is based on the principle of risk pooling, where premiums paid by participants are used to cover claims filed by individuals who have suffered losses. A key principle of insurance is the law of large numbers, which states that the larger the number of people insured suffer significant losses due to claims that exceed expectations. This allows insurance companies to better predict how many claims will be filed and how much they will cost.

Health insurance is a specialized form of insurance that focuses on providing financial protection against the cost of medical care. With the rising cost of health care. Health insurance is becoming increasingly important as a way to help individuals and families avoid the heavy financial burden of serious or chronic illness Schansberg, 2014).

Along with the development of medical technology and increasing life expectancy, the calculation of health insurance premiums has become increasingly complex. Health insurance must take into account the long-term possibilities associated with chronic diseases or diseases that require repeated treatment, such as stroke. One type of insurance that can be used as such coverage is term insurance.

Term insurance is a type of life insurance that provides protection for a specific period, with benefits paid only if the insured dies within that period. Term insurance is usually more affordable than whole life insurance, as it does not build cash value (Mehr & Cammack, 2003). The main advantage of term insurance is the relatively lower premium compared to other types of life insurance. Since it only provides protection for a limited period without any savings or investment elements, it is more affordable for individuals who only need temporary protection, such as to protect a family during critical times, for example when children still need education fees or the house is still in the mortgage period.

Term insurance premiums are determined based on several factors, including age, health, lifestyle, and contract duration. Since the premium must compensate for the risk of death over a certain period, the longer the policy term, the higher the premium that must be paid. Term insurance premiums tend to rise as the insured ages, especially if the policy is renewed after the initial protection period has expired. It is therefore ideal for individuals who need protection during certain phases of their lives, such as when children are still dependent on parents' income or when mortgage debt is outstanding.

2.2. Stroke Disease

Stroke is an acute disruption in the blood supply to the brain that causes brain cell death due to lack of oxygen. According to the World Health Organization (WHO), stroke is the leading cause of disability and death worldwide. Stroke is generally classified into two main types: ischemic stroke and hemorrhagic stroke. Ischemic stroke, which accounts for about 85% of total stroke cases, is caused by the blockage of blood flow to the brain by a blood clot or plaque in a blood vessel. In contrast, hemorrhagic stroke occurs when a blood vessel in the brain ruptures and causes bleeding in or around the brain tissue (Tazin, 2021).

Risk factors for stroke fall into two groups: modifiable risk factors and immutable risk factors. Immutable risk factors include age, gender, family history and race. Men have a higher risk of stroke than women, although women tend to have a higher risk of stroke in old age. Changeable risk factors include hypertension, diabetes, obesity, high cholesterol, smoking, and sedentary lifestyle. Controlling these risk factors, such as lowering blood pressure, changing diet, and increasing physical activity, has been shown to significantly reduce stroke risk.

Stroke treatment depends on the type of stroke experienced. For ischemic stroke, thrombolytic therapy such as the use of tissue plasminogen activator (tPA) administered within three to four and a half hours after stroke onset can dissolve the blood clot and restore blood flow. While in hemorrhagic stroke, treatment usually involves controlling high blood pressure, surgical intervention, or hematoma removal to reduce pressure on the brain. After the acute phase, stroke patients often require rehabilitation to restore motor, speech and cognitive functions. This rehabilitation involves physiotherapy, occupational therapy and speech therapy tailored to the patient's individual needs. A comprehensive rehabilitation program is proven to improve the quality of life and speed up the recovery of stroke patients.

2.3. Interest Rate

An interest rate is a fee that a borrower pays to a lender for the use of money over a certain period. In an economic context, interest rates play an important role as a means of monetary control, a determinant of investment value, as well as a driver of overall economic activity. Interest rates can also be defined as the percentage of the loan or deposit amount that is paid in interest over a period of time, usually expressed as an annual percentage known as the annual interest rate.

Interest rates in insurance are an important factor that affects the determination of premiums and reserves. Interest rates are used to calculate the present value of future payment obligations by insurance companies, and therefore, fluctuations in interest rates can affect the financial stability of insurance companies (Wariri, 2019).

2.4. Insurance Claims

An insurance claim is an application submitted by a policyholder to an insurance company to request payment or compensation for loss or damage incurred, in accordance with the provisions in the insurance policy. These claims can occur due to various events, such as accidents, illnesses, or natural disasters (Mehr & Cammack, 2003). The claims process is a core mechanism in the operations of insurance companies, where policyholders who experience losses or events covered by the policy can request payments to cover these losses. In the insurance industry, claims management is a very important factor to maintain policyholder trust and business continuity. An efficient and transparent claims process can enhance the reputation of an insurance company, while claims that are rejected or processed slowly can reduce policyholder confidence.

Insurance claims can be categorized based on the type of underlying insurance. Some common types of claims include life, health, vehicle, and property insurance claims. Life insurance claims are filed when the insured dies. The heirs or beneficiaries of the insured will receive payment from the insurance company in accordance with the coverage value agreed upon in the policy. Health insurance claims are filed when the insured suffers from an illness or accident that requires medical treatment. Meanwhile, vehicle and property insurance claims are submitted when the vehicle or property owned by the insured is damaged by an accident or natural disaster such as fire, flood, earthquake, and others. Insurance companies usually send an appraiser to assess the loss before deciding on the amount of compensation to be paid.

2.5. Markov Chain

Markov chain is a stochastic process that exhibits memoryless properties, where the probability of a future state depends only on the current state and not on previous events. One of the developments of the Markov Chain method is the Multistate Markov Chain method. Multistate Markov Chains are an extension of standard Markov Chains that allow them to be used with multiple states, making them suitable for modelling systems where a state can transition through multiple states over time. These models are particularly useful in fields such as healthcare, finance, and reliability engineering.

In insurance, Markov Chains are used to model the transition of an insured's health status or claim over a period of time. For example, in health insurance, a person can move from healthy to sick or from sick to dead, and the transition probabilities between these states can be used to calculate premiums. In addition, Markov Chains are also used to model the dynamics of life insurance portfolios. The insured can move from living to dead status, and the insurance company uses this transition probability to determine the value of claims to be paid to future beneficiaries.

In Markov Chains, there are several main components used in probabilistic modelling that describe stochastic processes, including the Transition Probability Matrix and the Transition Rate Matrix.

2.6. Transition Probability Matrix

A transition probability matrix is a mathematical representation of the probability of a system moving from one state to another in a stochastic process. They are often used in Markov Chain models to study the dynamics of transitions between states over time, helping to predict transition patterns with measurable probabilities. The transition probability matrix has several important properties that make it a very useful tool in the analysis of stochastic systems. The first property is that the row total probability always has a sum of one. The second property is that it is a stochastic matrix, which is a matrix whose every element is between zero and one. And the third property is the long-run transition probabilities. In some cases, if a matrix of transition probabilities is operated on iteratively (i.e. multiplying the matrix by itself), it can reach a stationary equilibrium or stationary distribution, i.e. a condition where the transition probabilities from one state to another no longer change over time. This gives an idea of how the system will behave in the long run.

Transition probability matrices are used in various fields of science to model stochastic processes. Some common applications of these matrices include the fields of insurance, genetics, and economics. In the context of health insurance or life insurance, transition probability matrices can be used to model the transition of health status from one category to another (for example, from healthy to sick, or from sick to dead). This matrix is used in the calculation of insurance premiums based on the probability of a person moving from one state to another over a period of time. The transition probability matrix can be illustrated as follows:

$$P(t) = \begin{bmatrix} P_{00}(t) & P_{01}(t) & \dots & P_{0j}(t) \\ P_{10}(t) & P_{11}(t) & \dots & P_{1j}(t) \\ \vdots & \vdots & \ddots & \vdots \\ P_{i0}(t) & P_{i1}(t) & \dots & P_{ij}(t) \end{bmatrix} \quad (1)$$

Description:

$P(t)$: Transition Probability Matrix.

$_{ij}(t)$: Coefficient of probability of state i to j transition in Transition Probability Matrix.

2.7. Transition Rate Matrix

The Transition Rate Matrix is a matrix that describes the rate of movement between states in a Markov Chain process. Each element in this matrix represents the rate of change or transition from one state to another, rather than a direct transition probability as in the Transition Probability Matrix. Transition rate matrices have a variety of uses in many fields, especially in stochastic systems modeling and continuous-time Markov process analysis. Some of the main uses of transition rate matrices include management engineering, biology and ecology, and queuing theory. The Transition Rate Matrix can be illustrated as follows:

$$M = \begin{bmatrix} -\mu_{00} & \mu_{01} & \cdots & \mu_{0j} \\ \mu_{10} & -\mu_{11} & \cdots & \mu_{1j} \\ \vdots & \vdots & \ddots & \vdots \\ \mu_{i0} & \mu_{i1} & \cdots & -\mu_{ij} \end{bmatrix} \quad (2)$$

Description:

M : Transition Rate Matrix.

$(t)(i \neq j)$: Transition rate coefficient of state i to j in the Transition Rate Matrix.

$_{ij}(t)(i = j)$: The transition rate negative coefficient and represents total transition rates from state i.

2.8. Eigenvalues and Vectors

Eigenvalue and eigenvector are fundamental concepts in linear algebra related to linear transformation of matrices. Eigenvalue is a scalar associated with a square matrix. The eigenvalue describes how much the eigenvector will stretch or shrink when applied by a matrix. For Eigen Vectors a non-zero vector whose transformative direction remains the same (only lengthened or shortened) when a linear transformation (in matrix form) is applied. An eigenvector is related to an eigenvalue, which measures the change in length of that vector after the transformation. To determine the eigenvalues and vectors of a square matrix, it is necessary to solve the following characteristic equation:

$$\det(A - \lambda I) = 0 \quad (3)$$

Description:

det : Matrix Determinant.

A : An n × n square matrix.

: Eigenvalue.

: Identity matrix of size n × n.

3. Materials dan Method

3.1. Materials

In this research, synthesized data will be used that is made close to the original data to calculate the insurance premium value of stroke patients. The data that will be used is about stroke patients who are categorized into 3 conditions at ABC Hospital. Data processing will be done through Microsoft Excel and Python.

The synthesized data created will describe the changes in the state of stroke patients with 3 different conditions from the time before entering the hospital and when leaving the hospital. For the calculation of a single premium, we will take the case of term life insurance with a term of one year and an interest rate of $\delta = 0.06$ or 6%, as well as the benefits received by the policyholder in the event of a claim or death of the policyholder are $Rp30{,}000{,}000.00$.

3.2. Method

Multistate Markov Chain is an extension of the standard Markov Chain that allows it to be used with multiple states, thus making it suitable for modelling systems where a state can transition through multiple states over time. The following are the steps that will be taken in applying the Multistate Markov Chain Method:

a). Determine the states that occur and the number of events that occur in the study.
b). Calculate the transition probability of each possible event transfer from the study.
c). Create a Transition Probability Matrix from the results of each calculated transition probability.
d). Calculate the amount of displacement between events affected by acceleration or transition rate. Determine the Transition Rate Matrix by reducing the diagonal matrix of the Transition Probability Matrix that has been obtained with the Identity Matrix.
e). Calculate the eigenvalue of the reduced matrix, then determine the Diagonal Matrix from the eigenvalue that has been obtained.
f). Determine the Eigen Vector Matrix of the Transition Rate Matrix.
g). Determine the new matrix from the inverse of the Eigen Vector Matrix.
h). Determine the present value of the annuity of each possible state transfer.
i). Determine the single premium value by summing up the total present value of the state transfer annuity, then multiplying it by the claim value if the policyholder dies.

4. Results and discussion

The results of this study include the process of determining the premium value for insurance for stroke patients from the ABC Hospital synthesis data using the Multistate Markov Chain method. For the first step, we will determine the conditions that occur and the number of events that occur in the study.

Table 2: Data synthesis of stroke patients at ABC Hospital

	→	End state of the patient		
		Stroke (State 1)	Healthy (State 2)	Died (State 3)
Initial state of the patient	Stroke (State 1)	66	19	24
	Healthy (State 2)	0	0	0
	Died (State 3)	0	0	0

The diagram can be seen in Figure 1 below:

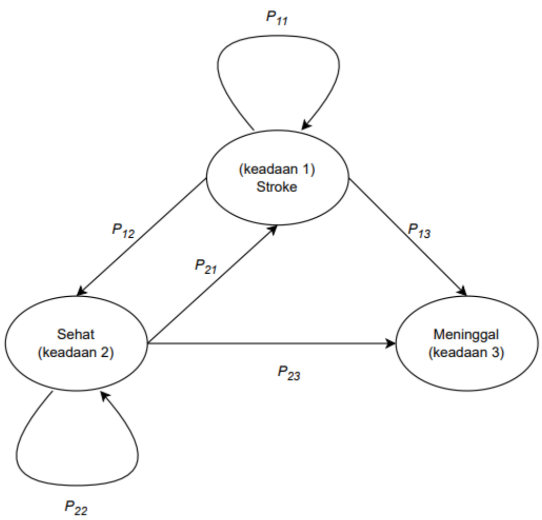

Figure 1: State transition probability diagram for stroke patients

For the second step, we will calculate the transition probability of each possible event transfer, and then create a Transition Probability Matrix from the results of each calculated transition probability. Define P_{ij} as the patient's transition probability from state to state. Then based on table 1 and figure 1 is obtained:

Table 3: Table of transition probabilities of moving each state

Transition probability of moving each state		
State 1 (Stroke)	State 2 (Healthy)	State 3 (Died)
$P_{11} = 66/109 = 0.606$	$P_{21} = 0$	$P_{31} = 0$
$P_{12} = 19/109 = 0.174$	$P_{22} = 0$	$P_{32} = 0$
$P_{13} = 24/109 = 0.220$	$P_{23} = 0$	$P_{33} = 0$

Then the value of the transition opportunity is loaded into the Transition Opportunity Matrix.

$$P(t) = \begin{bmatrix} 0.606 & 0.174 & 0.220 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{4}$$

For the next step, the Transition Rate Matrix will be formed by calculating the amount of displacement between events affected by acceleration or transition rate. The Transition Rate Matrix can be found by reducing the diagonal matrix of the Transition Probability Matrix with the Identity Matrix.

$$M = \begin{bmatrix} -0.394 & 0.174 & 0.220 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \tag{5}$$

The value of the transition probability function can be obtained if the eigenvalues and vectors of the Transition Rate Matrix or Matrix M are found. By solving equation (3), the eigenvalues of the Matrix are $\lambda_1 = -0.394$, $\lambda_2 = -1$, and $\lambda_3 = -1$, so that Matrix D is obtained as the Diagonal Matrix of the eigenvalue.

$$D = \begin{bmatrix} -0.394 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \tag{6}$$

For the next step, the Eigen Vector Matrix or Matrix A of the previously obtained Matrix M will be determined by solving equation (3).

$$A = \begin{bmatrix} 1 & -0.27597779 & -0.34124485 \\ 0 & 0.96116401 & 0 \\ 0 & 0 & 0.93997444 \end{bmatrix} \tag{7}$$

Next, a new matrix or Matrix C will be determined from the result of inversing the Eigen Vector Matrix or Matrix A.

$$C = \begin{bmatrix} 1 & 0.28712871 & 0.3630363 \\ 0 & 1.04040516 & 0 \\ 0 & 0 & 1.06385871 \end{bmatrix} \tag{8}$$

For the next step, the present value of the annuity of each possible state transfer will be calculated using the formula for calculating term annuities as follows:

$$\bar{A}'_{x:\bar{n}|} = \sum_{k=1}^{4} \frac{1}{(\delta - d_k)} \left(1 - e^{-(\delta - d_k)n}\right) a_{ik} c_{kj} \mu_{ij} \tag{9}$$

Description:

$\bar{A}'_{x:\bar{n}|}$: Present value of annuity.
δ : Interest rate.
d_k : Discount factor.
a_{ik} : The ikth element in the Eigen Vector Matrix
c_{kj} : The kjth element in the Eigen Vector Matrix Inverse.
μ_{ij} : The ijth element in the Transition Rate Matrix.

It is known that the term life insurance has a term of one year and has an interest rate of $\delta = 0.06$ or 6%, as well as the benefits received by the policyholder in the event of a claim or the policyholder dies is $IDR30{,}000{,}000.00$. The results of the calculation of the present value of each annuity are shown in Table 4 below:

Table 4: Calculation result of Present Value of Annuities at each state transfer

Present Value of Annuity at Each State Transfer	Calculation Result
$\bar{A}'_{x:\bar{n}\|11}$	-0.316690315
$\bar{A}'_{x:\bar{n}\|12}$	0.009354154
$\bar{A}'_{x:\bar{n}\|13}$	0.014953794
$\bar{A}'_{x:\bar{n}\|21}$	0
$\bar{A}'_{x:\bar{n}\|22}$	0
$\bar{A}'_{x:\bar{n}\|23}$	0
$\bar{A}'_{x:\bar{n}\|31}$	0
$\bar{A}'_{x:\bar{n}\|32}$	0
$\bar{A}'_{x:\bar{n}\|33}$	0

The results of the calculation of the present value of the annuity of each state transfer are then accumulated and multiplied by the predetermined benefit amount of $IDR30{,}000{,}000.00$, so that the result of the single premium paid is obtained as follows:

$$\bar{A}'_{x:\bar{n}|} = \big((-0.316690315) + (0.009354154) + (0.014953794)\big) \times Rp30{,}000{,}000.00$$

$$= (-0.29238) \times Rp30{,}000{,}000.00 = -IDR8{,}771{,}471.00$$

The single premium of a life insurance with a term of 1 year has an interest rate of $\delta = 0.06$ or 6%, and the amount of benefits received by the policyholder in the event of a claim or death of the policyholder by $IDR30{,}000{,}000{,}00$ is $IDR8{,}771{,}471.00$

5. Conclussion

Based on the results and discussion obtained, it is known that the single premium for life insurance with a term of one year has an interest rate of 6%, and benefits by $IDR30{,}000{,}000.00$ if a claim occurs or the policyholder dies from stroke using the Multistate Markov Chain Method with 3 conditions on the ABC Hospital synthesis data is equal to $IDR8{,}771{,}471.00$.

References

Becker, M. (2016). *Efficient extraction of musculoskeletal structures from multi-channel MR images* (Doctoral dissertation, University of Geneva, Switzerland).

Brusca, S. B., & Albert, M. A. (2023). Balancing the scales of adversity: a socioecological approach to reducing the global burden of stroke and cardiovascular disease. *The Lancet Neurology*, *22*(12), 1094–1096. https://doi.org/10.1016/S1474-4422(23)00386-1

Habib, S. S., Perveen, S., & Khuwaja, H. M. A. (2016). The role of micro health insurance in providing financial risk protection in developing countries-a systematic review. *BMC public health*, *16*, 1-24.

Mehr, R. I., & Cammack, E. (2003). *Principles of Insurance*.

Ravi. (2023). *Health insurance schemes in India-an overview*.

Schansberg, D. E. (2014). The economics of health care and health insurance. *The Independent Review*, *18*(3), 401-420.

Tazin, T., Alam, M. N., Dola, N. N., Bari, M. S., Bourouis, S., & Monirujjaman Khan, M. (2021). Stroke disease detection and prediction using robust learning approaches. *Journal of healthcare engineering*, *2021*(1), 7633381.

Wariri, O., Edem, B., Nkereuwem, E., Nkereuwem, O. O., Umeh, G., Clark, E., ... & Kampmann, B. (2019). Tracking coverage, dropout and multidimensional equity gaps in immunisation systems in West Africa, 2000–2017. *BMJ global health*, *4*(5), e001713.