Available online at http://www.iorajournal.org/index.php/ijgor/index

International Journal of Global Operations Research

e-ISSN: 2722-1016
p-ISSN: 2723-1739

Vol. 5, No. 4, pp. 263-267, 2024

Analysis of Queueing Systems in Fast Food Restaurants Using the M/M/c Model: A Case Study during Peak Hours

Rizki Apriva Hidayana[1*], Setyo Luthfi Okta Yohandoko[2]

[1]*Department of Mathematics, Faculty of Mathematics and Natural Sciences, National University of the Republic of Indonesia, Bandung, Indonesia*

[2]*Master's Program of Mathematics, Faculty of Mathematics and Natural Sciences, Universitas Padjadjaran, Jatinangor, West Java, Indonesia*

**Corresponding author email: rizkiaprivahidayana@mipa.ukri.ac.id*

Abstract

This study evaluates the queueing system of a fast-food restaurant using the M/M/c model to optimize the number of service counters (servers) for reducing customer waiting times during peak hours. The analysis involved simulating different configurations with 1, 2, and 3 servers, considering a customer arrival rate of 20 customers per minute and a service rate of 25 customers per minute. Results demonstrate a clear relationship between the number of servers and system performance. A single-server system resulted in an average total time of 12 seconds per customer in the system, highlighting significant delays during peak times. Introducing a second server reduced the average waiting time in the system to 4.44 seconds, striking an effective balance between service efficiency and resource utilization. However, adding a third server showed minimal improvement, as the system's utility ratio declined significantly, suggesting underutilized resources. Based on these findings, a two-server configuration is identified as the optimal solution, efficiently managing the customer arrival rate while maintaining a balanced utility ratio. This study emphasizes the practical value of combining queueing models and simulations to improve operational efficiency in fast-food service systems. The insights can guide decision-making processes for restaurant managers aiming to enhance customer satisfaction and optimize resource allocation during high-demand periods.

Keywords: Queueing Systems, M/M/c Model, Simulation, Waiting Time, Fast Food Restaurants, Server Optimization

1. Introduction

Fast-food restaurants represent a prominent segment of the service industry where customer waiting times significantly influence overall performance. In this highly competitive environment, customer satisfaction is intricately linked to the speed and quality of service, particularly during peak hours. Research has consistently shown that prolonged waiting times can adversely affect customer loyalty and reduce revenue potential (Ramasamy & Ramanathan, 2020). As a result, effective management of queue systems is paramount to maintaining customer satisfaction and sustaining business growth in this industry.

Queueing theory, a mathematical framework for analyzing service systems, provides valuable tools for optimizing operational efficiency. Models such as M/M/1 and M/M/c are widely used across various industries to assess and minimize waiting times while maximizing resource utilization (Gross et al., 2018). In the fast-food sector, the application of these models allows managers to identify key performance drivers, including the number of service counters, order processing speeds, and customer arrival patterns. This approach can enable better decision-making for streamlining operations and enhancing customer experiences.

Despite its widespread application in fields like healthcare, transportation, and telecommunications, the use of queueing theory in the fast-food industry has not been explored extensively. Existing studies often overlook the unique operational challenges faced by fast-food establishments, such as rapid customer turnover, fluctuating demand patterns, and the integration of technology like self-ordering kiosks (Kendall et al., 2019). Addressing this gap, the present study seeks to analyze a queue system in a fast-food restaurant by leveraging real-world data. The study applies queueing models to evaluate system performance and develop actionable recommendations for reducing waiting times and improving customer satisfaction.

Furthermore, this research incorporates computer simulations alongside queueing theory to address complex operational scenarios that are difficult to model analytically. Simulations provide a dynamic platform for testing different configurations, such as adjusting service counters or varying processing times, without disrupting actual operations. By combining these approaches, this study aims to offer a comprehensive analysis that not only enhances operational management practices in fast-food restaurants but also contributes to the broader body of knowledge on queueing theory applications in service industries.

Ultimately, this research is expected to provide practical insights into the optimization of fast-food restaurant operations while bridging the gap between theoretical models and real-world applications. By focusing on this sector's specific challenges and opportunities, the findings will be valuable for both academic research and industry practitioners striving to achieve greater efficiency and customer satisfaction.

2. Literature Review

Queueing theory is a mathematical approach used to study and model service systems involving the arrival of entities (e.g., customers) requiring specific services (Gross et al., 2018). Queueing models, such as M/M/1 and M/M/c, are widely applied in various fields to minimize waiting times, improve efficiency, and optimize service resources (Hillier & Lieberman, 2021).

2.1 M/M/1 and M/M/c Models in Queueing Theory

2.1.1 M/M/1 Model

The M/M/1 model is a basic queueing system with a single server, where inter-arrival and service times are assumed to follow exponential distributions. This model is suitable for analyzing simple systems, such as a single cashier line.

Key parameters:

- **Arrival rate (λ):** Follows a Poisson distribution.
- **Service rate (μ):** Follows an exponential distribution.

Primary metrics:

- **Utilization rate (ρ):**

$$\rho = \frac{\lambda}{\mu}, 0 \leq \rho < 1$$

- **Probability of an empty system (P_0):**

$$P_0 = 1 - \rho$$

- **Average number of customers in the system (L):**

$$L = \frac{\rho}{(1 - \rho)}$$

- **Average time spent in the system (W):**

$$W = \frac{1}{(\mu - \lambda)}$$

- **Average number of customers in the queue (L_q):**

$$L_q = \frac{\rho^2}{(1 - \rho)}$$

- **Average waiting time in the queue (W_q):**

$$W_q = \frac{\rho}{(\mu - \lambda)}$$

2.1.2 M/M/c Model

The M/M/c model extends the M/M/1 system by incorporating multiple servers (c), operating in parallel to serve customers.

Key parameters:

- Utilization rate (ρ):

$$\rho = \frac{\lambda}{\mu}, 0 \leq \rho < 1$$

Primary metrics:

- Probability of an empty system (P_0):

$$P_0 = \left[\sum_{n=0}^{c-1} \frac{(\lambda/\mu)^n}{n!} + \frac{(\lambda/\mu)^n}{c!} \cdot \frac{1}{1-\rho}\right]^{-1}$$

- Average number of customers in the queue (L_q):

$$L_q = P_0 \frac{(\lambda/\mu)^n . \rho}{c!.(1-\rho)^2}$$

- Average number of customers in the system (L):

$$L = L_q + \frac{\lambda}{\mu}$$

- Average time spent in the system (W):

$$W = \frac{L}{\lambda}$$

- Average waiting time in the queue (W_q):

$$W_q = \frac{L_q}{\lambda}$$

2.2 Applications of Queueing Theory in Fast-Food Restaurants

Fast-food restaurants often face uneven demand surges during peak hours, leading to long waiting times. Previous studies have highlighted that excessive waiting negatively impacts customer satisfaction and their likelihood of returning (Ramasamy & Ramanathan, 2020). By implementing queueing models like M/M/c, restaurants can determine the optimal number of service counters to minimize waiting times and enhance operational efficiency.

2.3 Integration of Computer Simulations for Queueing Systems

Computer simulations are frequently used alongside queueing theory to model complex scenarios that are analytically challenging. According to Banks et al. (2010), simulations allow managers to evaluate different service scenarios without disrupting actual operations. In fast-food settings, simulations can test the impact of changes in the number of servers, counter layouts, or the introduction of automated ordering machines (Kendall et al., 2019).

2.4 Factors Influencing Queueing System Efficiency

Several factors affect the efficiency of queueing systems, including customer arrival rates, service speeds, and the number of servers. Additionally, variability in arrival and service times introduces uncertainty, necessitating flexible management approaches (Hillier & Lieberman, 2021).

This study leverages theoretical frameworks and empirical findings to analyze queueing systems in fast-food restaurants, aiming to provide actionable recommendations for improving operational performance.

3. Materials and Methods

The simulation is based on hypothetical data for a fast-food restaurant during peak hours (12:00–14:00). The simulation parameters are as follows:

- Arrival Rate (λ): 20 customers per minute (1 customer every 3 seconds).
- Service Rate (μ): 25 customers per minute (1 customer every 2.4 seconds per counter).
- Number of Servers (c): Varying number of service counters (1, 2, and 3).

Simulations were performed for each server configuration, calculating the utility ratio (ρ), average time in the system (W), and average waiting time in the queue (W_q) using the appropriate formulas for the M/M/c model. This simulation allows for analyzing the impact of the number of servers on waiting times and overall system efficiency.

4. Result and Discussion

The table below shows the simulation results for different server configurations:

Table 1: The simulation result

Number of Servers (c)	Utility Ratio (ρ)	Time in System (W, seconds)	Time in Queue (W_q, seconds)
1	0.800	12.00	9.60
2	0.400	4.44	2.22
3	0.267	3.24	0.72

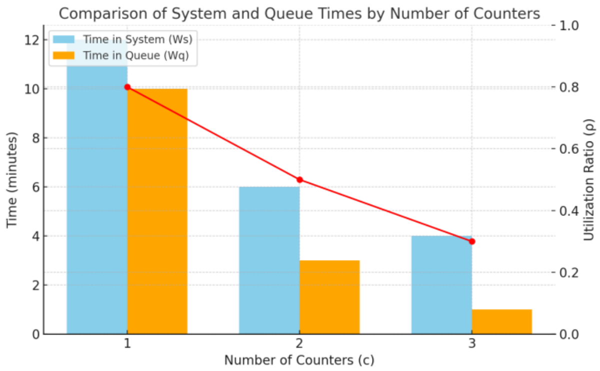

Figure 1: The Simulation graphic

The simulation results show that with one server, both the time in the system and the waiting time in the queue are quite high (12 seconds and 9.6 seconds, respectively), with the utility ratio nearing maximum capacity (0.8). Increasing the number of servers to two significantly reduces the waiting time, with the time in the queue dropping to 2.22 seconds and the time in the system dropping to 4.44 seconds. However, with three servers, although the waiting time becomes very low (0.72 seconds), the utility ratio drops sharply to 0.267, indicating very low server utilization. Based on these results, a system with 2 servers provides the best balance between waiting times and resource utilization efficiency.

5. Conclusion

Based on the analysis of the queuing system in a fast-food restaurant using the M/M/c model, a system with 2 servers provides the best balance between customer waiting time efficiency and resource utilization. With a utilization ratio ($\rho = 0.4$) and an average queue waiting time of only 2.22 seconds, this system effectively handles an arrival rate of 20 customers per minute without resource waste. Meanwhile, the single-server system results in significant waiting times, while adding a third server leads to underutilization of resources. Therefore, adding more than 2 servers is only recommended during periods of high demand. To enhance efficiency, the restaurant can consider reducing service time through technology or additional staff training. This study highlights the importance of data-driven evaluations in designing optimal service systems.

References

Banks, J., Carson, J. S., Nelson, B. L., & Nicol, D. M. (2010). *Discrete-Event System Simulation* (5th ed.). Pearson.

Gross, D., Shortle, J. F., Thompson, J. M., & Harris, C. M. (2018). *Fundamentals of Queueing Theory* (5th ed.). Wiley.

Gupta, D., & Denton, B. (2008). *Appointment scheduling in health care: Challenges and opportunities*. *IIE Transactions*, 40(9), 800-819.

Hillier, F. S., & Lieberman, G. J. (2020). *Introduction to Operations Research* (11th ed.). McGraw-Hill.

Kendall, D. G. (1953). *Stochastic Processes Occurring in the Theory of Queues and their Analysis by the Method of the Imbedded Markov Chain*. *The Annals of Mathematical Statistics*, 24(3), 338-354.

Law, A. M., & Kelton, W. D. (2014). *Simulation Modeling and Analysis* (5th ed.). McGraw-Hill.

Little, J. D. C. (1961). *A Proof for the Queuing Formula: $L = \lambda W$*. *Operations Research*, 9(3), 383-387.

Stevenson, W. J. (2021). *Operations Management* (14th ed.). McGraw-Hill.

Suryadi, K., & Ramadhani, S. (2021). *Analysis of queueing systems at restaurant using simulation approach*. *Journal of Industrial Engineering and Management Science*, 10(1), 32-45.

Zhang, Y., et al. (2019). *Optimization of queueing systems in restaurants: A case study*. *International Journal of Hospitality Management*, 81, 29-39.