



## Application of the Geometric Brownian Motion Model and Value at Risk Calculation on the Stock of PT Bank Tabungan Negara (Persero) Tbk

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### Abstract

The fluctuating nature of stock prices creates risks for investors, making quantitative methods essential for predicting price movements and estimating potential losses. This study applies the Geometric Brownian Motion (GBM) model to simulate the stock price dynamics of PT Bank Tabungan Negara (Persero) Tbk (BBTN) and calculates the Value at Risk (VaR) using the Monte Carlo simulation method. Daily closing price data from May 26 to September 26, 2025, were analyzed and confirmed to follow a normal distribution based on the Kolmogorov–Smirnov test. The results indicate a high prediction accuracy with a Mean Absolute Percentage Error (MAPE) of 7.95%. The estimated daily VaR for an initial capital of IDR 100,000,000 ranges from IDR 97,974 to IDR 114,045, corresponding to confidence levels between 80% and 99%.

**Keywords:** Geometric Brownian Motion, Value at Risk, Monte Carlo, stock.

### 1. Introduction

The capital market is one of the key instruments in the modern economy, serving as a platform for companies to raise funds and for investors to allocate their capital. Through the capital market, economic activity can grow more dynamically as funds flow from those with a surplus of capital to those in need of financing. However, the primary characteristic of the capital market lies in its inherent uncertainty and high stock price volatility. According to Sinha et al. (2024), stock price fluctuations are stochastic in nature and difficult to predict deterministically, thus requiring mathematical models capable of realistically representing stock price dynamics.

One of the most widely used approaches in stock price modeling is the Geometric Brownian Motion (GBM) model. This model assumes that stock price changes follow a continuous stochastic process with a log-normal distribution, ensuring that stock prices remain positive. Mensah et al. (2023) explain that GBM is particularly relevant because it is defined by two main parameters drift (average growth rate) and volatility (degree of uncertainty) which can be empirically estimated from historical data. Moreover, the GBM model serves as the foundational framework for the well-known Black-Scholes model, which is widely used in option pricing and other derivative instruments.

On the other hand, stock price forecasting is not only aimed at predicting future values but also at measuring the level of potential risk. One of the most commonly used risk measures in financial institutions is Value at Risk (VaR). According to Morkūnaitė (2024), VaR estimates the maximum potential loss at a given confidence level over a specified time horizon. This approach can be implemented through parametric methods, historical simulation, or Monte Carlo simulation. The Monte Carlo method is often preferred because it effectively captures the complex probability distributions of investment returns and closely reflects real market conditions.

Several previous studies have combined the Geometric Brownian Motion model with the Monte Carlo method to estimate both risk and stock prices. For example, Nguyen and Tran (2022) demonstrated that the GBM–Monte Carlo combination provides more accurate stock price estimates for the Vietnamese stock market compared to the traditional ARIMA model. Similarly, Ghosh and Mitra (2023) found that the GBM approach effectively captures stock price dynamics in India's financial sector, particularly during periods of high market volatility. In Indonesia, Putri and Widodo (2022) applied GBM to model the stock movements of Bank Mandiri and found that the model effectively represented medium-term upward price trends. Furthermore, Rahman and Fadillah (2023) combined GBM with Monte

Carlo simulation to calculate VaR for energy sector stocks, showing strong accuracy in predicting portfolio risk at a 95% confidence level.

In the context of the Indonesian capital market, the banking sector is one of the most dominant sectors on the Indonesia Stock Exchange (IDX). Banking stocks, especially those belonging to state-owned banks, often exhibit high volatility in line with macroeconomic fluctuations and monetary policy adjustments. Muslimin (2021) found that the stock price of PT Bank Tabungan Negara (Persero) Tbk (BBTN) tends to experience significant fluctuations following changes in interest rates and Bank Indonesia's liquidity policies. Therefore, applying the GBM model to project BBTN's stock price and calculating VaR through Monte Carlo simulation are essential steps toward providing a quantitative insight into potential risks and investment opportunities within this sector.

Through this study, it is expected that a more comprehensive understanding can be obtained regarding the application of stochastic models in stock risk analysis in the Indonesian capital market, particularly within the banking sector, which plays a crucial role in maintaining national economic stability.

## 2. Literature Review

### 2.1. Stock Return

Stock return is the return an investor receives from their stock investment. According to Tandelilin (2010), return is one of the factors that motivates investors to invest and is also the reward for the investor's courage in bearing the risk of their investment. Stock returns can be calculated using the natural logarithm formula as follows:

$$r(t) = \ln\left(\frac{S(t)}{S(t-1)}\right) \quad (1)$$

where  $r(t)$  represents the stock return in period  $t$ ,  $S(t)$  represents the stock price in period  $t$ , and  $S(t-1)$  represents the stock price in the previous period. The use of natural logarithms in return calculations has advantages because it produces time-additive returns and can capture continuous price changes (Hull, 2018). The average stock return over a specific period can be calculated using the formula:

$$\hat{\mu} = \frac{1}{n} \sum_{t=1}^n r(t) \quad (2)$$

where  $n$  is the number of observations.

### 2.2. Stock Volatility

Volatility is a measure of the dispersion of stock returns, reflecting the level of uncertainty or risk of an investment. According to Bodie et al. (2014), high volatility indicates that stock prices tend to fluctuate within a wide range, while low volatility suggests that stock prices are relatively stable. Volatility is calculated as the standard deviation of stock returns:

$$\hat{\sigma} = \sqrt{\frac{1}{n-1} \sum_{t=1}^n (r(t) - \hat{\mu})^2} \quad (3)$$

where  $\hat{\sigma}$  is the volatility estimate,  $r(t)$  is the return in period  $t$ , and  $\hat{\mu}$  is the average return.

### 2.3. Normality Test

The normality assumption is an important prerequisite for applying the Geometric Brownian Motion model and calculating Value at Risk. According to Gujarati & Porter (2009), the normality test is used to determine whether the sample data comes from a normally distributed population. The Kolmogorov-Smirnov test is one of the commonly used methods for testing the normality of data. The hypotheses in this test are:

$H_0$ : The data is normally distributed

$H_1$ : The data is not normally distributed

The Kolmogorov-Smirnov test statistic is:

$$D_{count} = \max |S(x) - F_0(x)| \quad (4)$$

where  $S(x)$  is the sample cumulative distribution function and  $F_0(x)$  is the theoretical cumulative distribution function being tested.

## 2.4. Stochastic Process

A stochastic process is defined as a collection of random variables indexed by time, denoted as  $\{X(t), t \in T\}$ , where  $t$  represents the time parameter and  $X(t)$  represents the value of the process at time  $t$  (Ross, 2014). Stock price movements are an example of a stochastic process in finance because stock prices change randomly over time. This random nature is caused by various factors such as unpredictable economic and political conditions, and market sentiment (Fabozzi et al., 2010).

## 2.5. Brownian Motion

Brownian motion, is a continuous-time stochastic process used to describe the random movement of a variable. According to Hull (2018), a stochastic process  $\{W(t), t \geq 0\}$  is called standard Brownian motion if it satisfies the following characteristics:

- 1) Initial value:  $W(0) = 0$
- 2) Independent increments: For non-overlapping time intervals, the changes in the process are independent.
- 3) Normal distribution:  $W(t) - W(s) \sim N(0, t - s)$  for  $0 \leq s < t$
- 4) Continuity: The process path is continuous with probability

Brownian Motion with drift  $\mu^*$  and variant  $\sigma^2$  can be expressed:

$$B(t) = \mu^*(t) + \sigma W(t) \quad (5)$$

where  $W(t)$  is standard Brownian Motion.

## 2.6. Lemma Itô

Lemma Itô, developed by Kiyoshi Itô in 1951, is a fundamental tool in stochastic calculus. If  $x$  follows an Itô process:

$$d(x) = a(x, t)dt + b(x, t)dW(t) \quad (6)$$

and if there exists a differentiable function  $G(x, t)$ , then  $G$  follows:

$$dG = \left( \frac{\partial G}{\partial x} a + \frac{\partial G}{\partial t} + \frac{1}{2} \frac{\partial^2 G}{\partial x^2} \right) dt + \frac{\partial G}{\partial x} bdW(t) \quad (7)$$

This lemma is important in the derivation of financial asset pricing models, including the Black-Scholes model (Øksendal, 2013).

## 2.7. Geometric Brownian Motion (GBM)

Geometric Brownian Motion is a widely used mathematical model for modeling stock prices. This model was first introduced in a financial context by Samuelson (1965) and was later further developed by Black & Scholes (1973). The GBM model assumes that the logarithm of stock returns is normally distributed and that stock prices are always positive. The stochastic differential equation for GBM is:

$$dS(t) = \mu S(t)dt + \sigma S(t)dW(t) \quad (8)$$

The solution to this equation is:

$$S(t) = S(0) \exp \left( \left( \mu - \frac{\sigma^2}{2} \right) t + \sigma W(t) \right) \quad (9)$$

In discrete implementation, the GBM model can be written as:

$$S(t) = S(t-1) \exp \left( \left( \hat{\mu} - \frac{\hat{\sigma}^2}{2} \right) \Delta t + \hat{\sigma} \varepsilon \sqrt{\Delta t} \right) \quad (10)$$

where  $\varepsilon$  is a standard normal distribution random number.

## 2.8. Prediction Accuracy

Evaluating the accuracy of a prediction model is an important step in model validation. Mean Absolute Percentage Error (MAPE) is one of the commonly used metrics for measuring prediction accuracy. Lewis (1982) categorized accuracy levels based on MAPE values:

- (a) Highly accurate:  $MAPE \leq 10\%$
- (b) Good:  $10\% < MAPE \leq 20\%$
- (c) Reasonable:  $20\% < MAPE \leq 50\%$
- (d) Inaccurate:  $MAPE > 50\%$

MAPE is calculated using the formula:

$$MAPE = \frac{1}{n} \sum_{t=1}^n \frac{|S(t) - \hat{S}(t)|}{S(t)} \times 100\% \quad (11)$$

where  $S(t)$  is the actual value and  $\hat{S}(t)$  is the predicted value.

## 2.9. Value at Risk (VaR)

Value at Risk (VaR) Value at Risk (VaR) is a risk measure that quantifies the maximum potential loss on an investment within a specific time period and at a certain confidence level. The VaR concept was first popularized by J.P. Morgan in the 1990s thru the RiskMetrics system (Jorion, 2007). Mathematically, VaR with a confidence level of  $(1 - \alpha)$  for a period of  $t$  days can be expressed as:

$$VaR_{(1-\alpha)}(t) = V_0 R^* \sqrt{t} \quad (12)$$

where  $V_0$  is the initial investment capital and  $R^*$  is the  $\alpha$ -quantile value of the return distribution.

### 2.9.1. Monte Carlo Simulation Method for VaR

Monte Carlo simulation is a numerical method that uses random sampling to solve mathematical or statistical problems. In the context of VaR, this method simulates by randomly generating return scenarios based on historical parameters (Glasserman, 2004). The advantages of the Monte Carlo method include:

- 1) Flexibility in accommodating various distributions.
- 2) Ability to handle complex portfolios.
- 3) High accuracy with a sufficient number of simulations.

## 3. Material and Methods

### 3.1. Material

This study uses daily closing price data for PT Bank Tabungan Negara (Persero) Tbk (BBTN) shares, obtained from the id.investing.com website, for the period from May 26, 2025, to September 26, 2025. The total amount of data used is 83 trading days. The data is divided into two parts:

- 1) In-sample data, comprising 39 observations (May 26 – July 17, 2025) used to estimate the parameters of the Geometric Brownian Motion (GBM) model, namely the mean (drift,  $\mu$ ) and volatility ( $\sigma$ ).
- 2) Out-sample data, comprising 44 observations (July 18 – September 26, 2025) used as model validation data to test the accuracy of the predictions.

In addition to stock price data, this research also uses Python software as a numerical computing tool to assist in the calculation process, Monte Carlo simulation, and result visualization.

### 3.2. Methods

This research uses a quantitative method with an explanatory research type, as it aims to prove the cause-and-effect relationship or the mutually influencing relationship between the variables being studied (Istijanto, 2006). In this context, the relationship between stock prices, return rates, volatility, and risk (VaR) is analyzed mathematically thru stochastic modelling. The quantitative methods used are the Geometric Brownian Motion (GBM) stochastic modelling to describe stock price movements and Monte Carlo Simulation to calculate Value at Risk (VaR) as a measure of financial risk.

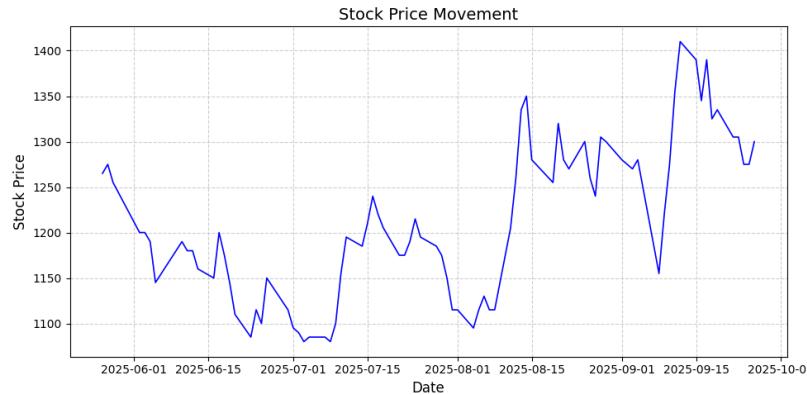
## 4. Results and Discussion

### 4.1. Characteristics of PT Bank Tabungan Negara (Persero) Tbk Stock Prices

Figure 2 illustrates the movement of PT Bank Tabungan Negara (Persero) Tbk (BBTN) closing stock prices during the period from May 26 to September 26, 2025. Throughout this period, the stock price exhibited notable fluctuations, reaching its lowest level of IDR 1,080 on July 3 and July 8, 2025, and peaking at IDR 1,410 on September 12, 2025. Overall, the price remained within the range of IDR 1,100–IDR 1,400, showing a generally upward trend toward the end of the observation period.

The subsequent period was marked by a recovery phase (rebound) that pushed the stock price back up into the IDR 1,200–IDR 1,400 range toward the end of the observation window. This pattern indicates that BBTN's stock price does not move linearly but is instead influenced by market fluctuations reflecting both external and internal factors of the company.

For the purpose of quantitative analysis, the closing price data of BBTN were divided into two segments: in-sample data for model training and out-sample data for model testing. This division aimed to evaluate the accuracy and reliability of the forecasting method when compared against actual data.



**Figure 1:** Movement of Closing Stock Prices of PT Bank Tabungan Negara (Persero) Tbk

#### 4.2. In-Sample Return Data

The in-sample return data were calculated using Equation (1). A total of 83 observations were used as in-sample data. The descriptive statistics of the in-sample return data are presented in Table 1 below:

**Table 1: Descriptive Statistics of In-Sample Return Data**

Statistika Deskriptif	<i>Return in Sample</i>
Mean	0.000333
Median	-0.004253
Standard Deviation	0.030216
Minimum	-0.10276
Maximum	0.077625
Number of Observations	82

Based on Table 1, it can be observed that the maximum return of BBTN's in-sample data is 0.077625, while the minimum return is -0.10276. The positive average return indicates the potential for gains from investing in this stock. Meanwhile, the mean return and standard deviation of the in-sample data are used as parameters in constructing the Geometric Brownian Motion (GBM) model. Before applying the GBM model, a normality test was conducted using the Kolmogorov–Smirnov method at a 5% significance level ( $\alpha = 0.05$ ) with the assistance of Python software.

**Tabel 2: Kolmogorov-Smirnov Test of In-Sample Stock Returns**

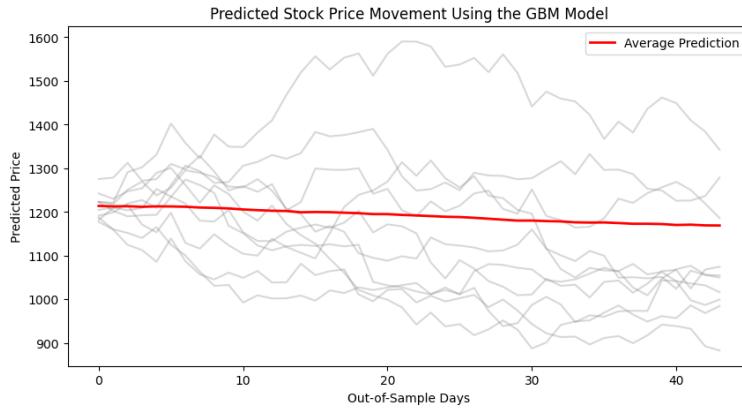
Kolmogorov-Smirnov Test	
$D_{statistic}$	0.13854
$p - value$	0.0779

According to Table 2, the results of the Kolmogorov–Smirnov test show that the  $p - value = 0.0779$ , which is greater than the significance level ( $\alpha = 0.05$ ). In addition, the calculated test statistic  $D_{statistic} = 0.13854$  is smaller than the critical value  $D_{table(\alpha, 82)} = 0.15019$ . Therefore, the null hypothesis ( $H_0$ ) is accepted, indicating that the in-sample return data follow a normal distribution. Consequently, these data are considered suitable for modeling using the Geometric Brownian Motion (GBM) framework.

#### 4.3. Stock Price Prediction Using the Geometric Brownian Motion Model

The parameters of the Geometric Brownian Motion (GBM) model include an average in-sample return of  $-0.001061$  and a return volatility of  $0.23896$ . Thus, the GBM model for BBTN's stock price, with a time step of one trading day, can be formulated as follows:

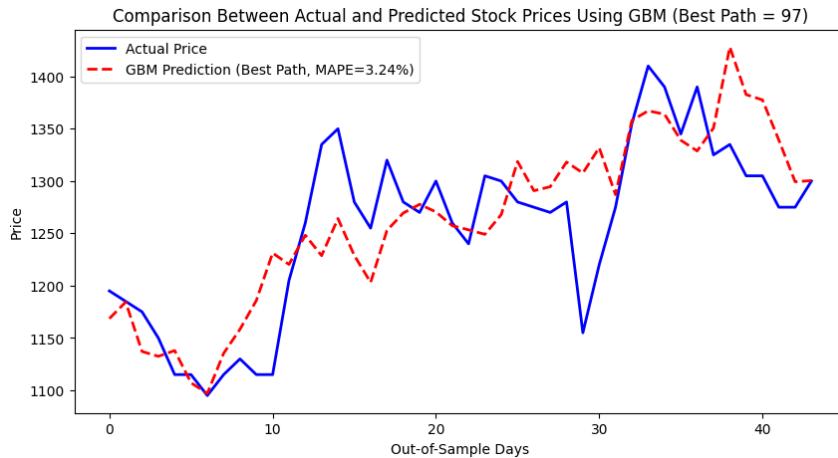
$$S(t) = S(t-1) \exp \left( \left( -0.001061 - \frac{(0.023896)^2}{2} \right) (1) + (0.023896 \varepsilon \sqrt{1}) \right)$$



**Figure 2:** Predicted Stock Price Movement Using the GBM Model

Prediction was carried out for a two-month period (44 trading days) using 2,000 simulation runs, resulting in 2,000 simulated price paths. The visualization of the predicted stock price movements is presented in Figure 3.

Based on the GBM prediction results, the average Mean Absolute Percentage Error (MAPE) obtained is 7.95%. Among the 2,000 simulated price paths, the smallest MAPE value recorded is 3.24%. According to the MAPE accuracy criteria, the prediction using the GBM model falls under the “very good” accuracy category, as all MAPE values are below 10%. A comparison between the actual stock prices and the predicted price path with the smallest MAPE is illustrated in Figure 4.



**Figure 3:** Comparison Between Predicted and Actual Stock Prices

#### 4.4. Value at Risk (VaR) Calculation Using Monte Carlo Simulation

The Monte Carlo Value at Risk (VaR) was calculated based on the predicted returns generated from the GBM model, consisting of 43 data points with an average return of  $-0.000874$  and a volatility of  $0.000761$ . Since the returns were assumed to follow a normal distribution, a Kolmogorov–Smirnov normality test was conducted at a 5% significance level ( $\alpha = 0.05$ ).

**Table 3:** Kolmogorov–Smirnov Test for Predicted Stock Returns

Kolmogorov–Smirnov Test	
<i>D</i> _hitung	0.10806
<i>p</i> – value	0.6571

The results of the Kolmogorov–Smirnov test for the predicted stock returns (Table 3) show that the  $p$  – value  $> \alpha$  and the calculated statistic  $D_{statistic} < D_{table(\alpha, 43)} = 0.20740$ . Thus, the null hypothesis ( $H_0$ ) is accepted, indicating that the predicted BBTN stock returns follow a normal distribution and are suitable for VaR estimation.

The simulation of returns was performed by generating 43 random data points based on the predicted mean return (0.00036) and volatility (0.01393), repeated 1,000 times. From these simulations, the maximum losses ( $\alpha$ -quantiles) and corresponding VaR values were computed for confidence levels of 80%, 90%, 95%, and 99% over a one-day period, with an initial capital of *IDR* 100,000,000. The average maximum losses and VaR values from the 1,000 simulations are presented in Table 4.

**Table 4:** Values of  $R^*$  and  $VaR$ 

Confidence Level	$R^*$	$VaR(\%)$	$VaR (Rp)$
80%	–0.00098	0.097974	–97,973.73
90%	–0.001033	0.103339	–103,338.55
95%	–0.001079	0.107905	–107,905.35
99%	–0.00114	0.114045	–114,044.64

Based on the VaR results in Table 4, it can be interpreted that if an investor invests *IDR* 100,000,000 in PT Bank Tabungan Negara (BBTN) stock, then at a 99% confidence level, the average VaR is 0.1140%, meaning that the maximum potential loss in the next trading day is approximately *IDR* 114,045. In other words, there is a 1% probability that the daily loss could reach *IDR* 114,045 or more. At a 95% confidence level, the maximum potential daily loss is around *IDR* 107,905, while at 90% and 80% confidence levels, the potential losses are approximately *IDR* 103,339 and *IDR* 97,974, respectively. These results indicate that the higher the confidence level used, the greater the potential risk exposure that investors must be prepared to bear.

#### 4.5. General Discussion

The results of this study demonstrate that the Geometric Brownian Motion (GBM) model is capable of accurately representing the dynamic behavior of PT Bank Tabungan Negara (Persero) Tbk (BBTN) stock prices. This is evident from the average Mean Absolute Percentage Error (MAPE) of 7.95%, which falls under the “very good” category (MAPE  $< 10\%$ ). This finding indicates that the assumption of a log-normal stochastic process underlying the GBM model aligns well with the characteristics of BBTN’s stock price data during the study period.

The estimated drift ( $\mu$ ) and volatility ( $\sigma$ ) parameters derived from the in-sample data reflect the short-term trend direction and level of uncertainty in stock price movements. The relatively small drift value suggests that BBTN’s stock price growth tends to be stable without sharp surges, while the relatively high volatility indicates that the stock remains influenced by external factors such as interest rate policies and macroeconomic conditions. This pattern is consistent with the findings of Muslimin (2021), who reported that the stock prices of state-owned banks listed on the Indonesia Stock Exchange (IDX) are sensitive to changes in monetary policy and market liquidity.

Furthermore, the Kolmogorov–Smirnov normality test results for both the in-sample and predicted data show that the stock return distributions satisfy the normality assumption. This supports the validity of the GBM model, as one of its core assumptions is that log-returns of stock prices follow a normal distribution with a specific mean and variance.

The Monte Carlo simulation results reveal that the Value at Risk (VaR) increases with higher confidence levels. At the 99% confidence level, the VaR value of 0.1140% indicates a maximum potential daily loss of approximately *IDR* 114,045 for an initial investment of *IDR* 100,000,000. This finding aligns with investment risk theory, which states that the higher the confidence level, the greater the potential maximum loss an investor must anticipate (Morkūnaitė, 2024). These results are also consistent with Rahman and Fadillah (2023), who found that the VaR values of energy sector stocks in Indonesia increase proportionally with the confidence levels used in Monte Carlo simulations.

Overall, this study demonstrates that the GBM model combined with the Monte Carlo approach can serve as a practical tool for measuring risk and forecasting stock prices in the Indonesian capital market, particularly in the banking sector. The model effectively captures the stochastic behavior of stock prices and provides a realistic estimation of potential losses.

However, it is important to note that GBM has limitations, as it assumes constant volatility and does not account for jumps or extreme price movements that often occur under unstable market conditions. Ghosh and Mitra (2023) suggested that the GBM model could be enhanced by incorporating stochastic volatility or jump-diffusion approaches to produce more adaptive estimates under real-world market fluctuations. Hence, future research is encouraged to develop hybrid models capable of capturing more complex stock price dynamics.

In conclusion, the application of GBM and Monte Carlo simulation to BBTN stock provides empirical insights into how stochastic modeling approaches can be utilized in market risk analysis within the Indonesian capital market context, while also opening opportunities for the development of more accurate models to support future investment decision-making.

## 5. Conclusion

Based on the research findings regarding the application of Geometric Brownian Motion (GBM) and the calculation of Value at Risk (VaR) on the shares of PT Bank Tabungan Negara (Persero) Tbk (BBTN), it can be concluded that the GBM model is able to accurately describe the price movement of BBTN shares. The Kolmogorov-Smirnov normality test shows that the return data is normally distributed, making it suitable for modelling. Stock price prediction using the GBM model has an excellent accuracy rate with an average MAPE value of 7.95%, which is below 10%, thus falling into the highly accurate forecasting category. The results of the Monte Carlo simulation show that the predicted returns are also normally distributed, making them valid for risk estimation. The daily VaR calculation for a capital of IDR100,000,000 results in a maximum potential loss of IDR97,974 at an 80% confidence level to IDR114,045 at a 99% confidence level. The higher the chosen confidence level, the greater the potential loss that investors must bear. Thus, the combination of the GBM model and Monte Carlo simulation can be used as an effective quantitative method for projecting stock prices and measuring investment risk, particularly for investors focused on high-volatility banking stocks like BBTN.

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